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PRE-SERVICE MATHEMATICS TEACHERS' KNOWLEDGE DEVELOPMENT AND BELIEF CHANGE WITHIN A TECHNOLOGY-ENHANCED MATHEMATICS COURSE

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ABSTRACT

With the adoption of Common Core State Standards, mathematics teachers have been expected to emphasize conceptual understanding as much as procedural and computational fluency in their teaching. Development of a sound mathematical content knowledge during their teacher education could enable mathematics teachers to meet this expectation. The infusion of technology into the domain of mathematics education has also modified the nature of mathematical content knowledge. Regarding the expectations and changes in the nature of mathematical content knowledge, in this dissertation, I examined the influence of Geometer's Sketchpad (GSP) on pre-service middle grade mathematics teachers' Specialized Content Knowledge (SCK); how their beliefs about mathematics, teaching and technology affected their content development process; and the impact of a technology-enhanced geometry course on Technological Content Knowledge (TCK). These research routes resulted in three manuscripts to be submitted to high impact journals in the field of mathematics teacher education.

Two main theoretical frameworks guided the operationalization of the constructs under investigation: 1) Mathematical Knowledge for Teaching (MKT) (Ball, Thames, & Phelps, 2008); and 2) Technological Pedagogical Content Knowledge (TPACK) (Koehler & Mishra, 2005). With respect to the MKT framework, SCK was defined as the mathematical knowledge a teacher would utilize while answering a student's unexpected why question about a procedure highlighted, while making an attempt to understand a student's mathematical error, and while making sense of an unusual student procedure for a given task or problem. To categorize teachers' beliefs, I utilized Ernest's (1989)

categorization of beliefs about mathematics, Kuhs and Ball's (1986; cited in Thompson, 1992) framework for beliefs about teaching, and Chen's (2011) framework for beliefs about technology. Regarding TPACK framework, I defined TCK as the technology knowledge pertaining to GSP, awareness of its affordances and limitations while solving an open geometry problem.

A case study approach was used as the methodology for the study. 16 pre-service middle grade mathematics teachers who enrolled in a graduate geometry course in the fall semester of 2013 were the participants of the study. According to varieties in their SCK and beliefs at the beginning of the study, six of them were selected as focal participants who were interviewed three times during the semester. Task-based interviews, questionnaires, course artifacts, classroom observations, and a pre-post MKT assessment were the main data sources. Corbin and Strauss' (1998) open coding strategy, theme analysis and pattern matching (Yin, 2008), and narrative inquiry (Clandinin & Connely, 1996) were used within the analytical techniques.

Findings from this study showed pre-service teachers' common content knowledge development, availability of instructional opportunities to investigate their and other pre-service teachers' mathematical errors, and to justify their mathematical reasoning were factors influencing their SCK development. While GSP was influential for content knowledge development, teachers' views and beliefs about technology determined the level of their gains from the software. Data also allowed me to generate an analytical framework to evaluate pre-service teachers' TCK pertaining to GSP. The administration of this framework on data from three geometry tasks showed the necessity of instructional guidance to investigate the affordances of the software in order to effectively use it as a problem-solving tool. Regarding findings in this dissertation, future research would focus on the study of SCK development with a higher number of participants, and would evaluate course activities that are designed to accelerate mathematics teachers' TCK improvement with respect to the framework developed.

DEDICATION

This dissertation is dedicated to the friendship and memory of Ismail Arı. He was a doctoral candidate in Bogazici University who became a role model for me in terms of academic endeavor, perseverance and love for learning before I started for my graduate studies in 2008. He was very hardworking and successful in his field, full of energy to beautify his environment with social and joyful projects, until the darkness spread out to his life. I hope he is in the light now.

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CHAPTER ONE – INTRODUCTION

According to the *Professional Standards for Teaching Mathematics (PSTM)* by the National Council of Teachers of Mathematics (NCTM) (1989), prospective mathematics teachers have been expected to 1) be exposed to good mathematics teaching, 2) know mathematics and curriculum, 3) know students and their learning, 4) know mathematical pedagogy, 5) develop a mathematical identity for teaching mathematics, and 6) be open to life-long learning for professional development. Within these standards, support for the development of mathematical content knowledge, which can be defined as the knowledge of mathematical concepts, procedures, their connections, and their multiple representations, was highlighted as an essential component of teacher education programs. Another central goal of teacher education has been to help mathematics teachers connect mathematics occurring in school curriculum to advanced mathematics.

Principles and Standards for School Mathematics (*PSSM*) (NCTM, 2000), introduced in the following years, depicted mathematics as a discipline connected to other subjects and to daily life. Moreover, constructivist theory has dominated as a foundation for students' effective mathematics learning. Another central point in *PSSM* was the emphasis on technology integration for mathematics instruction. Teachers have been expected to be knowledgeable about technologies and how to integrate them in order to facilitate student learning.

Regarding both *PSTM* (NCTM, 1989) and *PSSM* (NCTM, 2000), NCTM has envisioned that high-quality mathematics instruction should be accessible to all students. In order for this vision to become a reality, mathematics teachers need to have a deep understanding of mathematical content as well as the ability to make pedagogical decisions that consider the diversified backgrounds, needs, abilities, interests of a new generation of students. In addition, it is necessary for mathematics teachers to envision a curriculum consisting of coherent mathematical activities and topics across grades, to approach mathematical topics from different perspectives, to represent them in different ways, to assess their understanding, and to visualize their learning trajectories. Approaching student differences in this way enables each student to learn mathematical concepts according to their personal experiences.

The development of *Common Core State Standards* (CCSSM) for mathematics (CCSSI, 2010) has further accelerated the need for mathematics teachers who has strong mathematics background with deep conceptual understanding (Porter, Hwang & Yang, 2011). The preparation of prospective mathematics teachers with strong mathematical content knowledge has become even more crucial considering that 45 states agreed to institute CCSSM in their schools (Goertz, 2010). Mathematics teacher educators and teacher education programs have adapted these new standards in order to prepare prospective teachers to reach these ambitious expectations.

CCSSM for mathematics is similar to the *PSSM* (NCTM, 2000) in that the CCSSM also emphasizes logic, reasoning, conjecturing, and mathematical argumentation for learning mathematics. The main goal for CCSSM has been to increase the coherency and focus within mathematics curriculum and to get students to achieve a deep understanding of mathematical topics. CCSSM in addition to the *PSSM* (NCTM, 2000) informed teachers and teacher educators about the amount and depth of content knowledge pre-service teachers (PSTs) have been supposed to acquire throughout their teacher education programs. This guidance was given in order to make sure that PSTs, on completion of their programs, would be capable of providing high-quality instruction to their students. With the emergence of CCSSM for mathematics, it is not only necessary for teachers to know how to reason with models abstractly and quantitatively, but they equally need to know how to create models with mathematics for scientific phenomena.

Mathematical Preparation of Mathematics Teachers

In response to the changes in what and how teachers should be teaching mathematics in K-12 schools, various organizations within the mathematics and mathematics education communities proposed recommendations and guidelines for teacher education programs and for the mathematical preparation of teachers. One such report was *Crossroads in Mathematics* (Cohen, 1995), which provided recommendations about the quality, amount, and structure of mathematics courses for teacher preparation programs. This report targeted the improvement of introductory collegiate mathematics courses for students completing bachelor degrees for all majors. It recommended mathematics departments offer foundation courses to provide students opportunity to develop the missing prerequisite knowledge required to take advanced mathematics courses, such as discrete mathematics, advanced algebra, and real analysis. In addition, the report made recommendations for instructional strategies to be relevant to students' lives and to include hands-on activities. However, lecturing and direct instruction were still mentioned as viable methods in circumstances where the course content was very abstract. Even though the report underlined the importance of pedagogy as much as subject-matter for undergraduate mathematics courses, it presented the standards for pedagogy and content separately. While these recommendations were put forward for all majors taking undergraduate mathematics, the same documents expected mathematics teachers to have the same coursework as the mathematics majors while preparing themselves for their profession. Prospective mathematics teachers in all levels were to develop mathematical knowledge beyond the level that they will teach in school in order to affect both strategies and substance of the instruction and to foster students' mathematical reasoning.

The emphasis on prospective mathematics teachers' coursework beyond the level they will teach proposed in the *Crossroads* report continued within the *Mathematical Education of Teachers* (MET) report (CBMS, 2001). With the *MET* report, the *Conference Board of the Mathematical Sciences* (CBMS) recommended 21 hours of advanced mathematics courses with pedagogy courses and field experience for the preparation of middle school mathematics teachers. The same document also expected middle school mathematics teachers to be mathematics specialists, and to know high school mathematics curriculum well.

Additionally, the *MET* report recommended teacher education programs offer capstone courses for teachers designed to help PSTs make connections between elementary school mathematics and high school mathematics content. This was recommended due to the fact that PSTs started to student teach during the last year in the program, and took pedagogy courses during the first two years, thus causing pedagogies learned within the program to become theoretical and loosely connected to practice (Lai, McCallum & Soto-Johnson, 2011). In the *MET* report, mathematics methods courses were described as the courses that focused on the integration of mathematics content and knowledge of students' learning. In the same report, the capstone courses required during the senior year were designed to help PSTs connect advanced mathematics courses concepts with middle school mathematics concepts. The same standards for undergraduate mathematics courses that were recommended by the *MET* report remained in the *Beyond Crossroads* report (Blair, 2006) with an emphasis on their connection to NCTM principles and standards (2000). More recently, the *Foundations for Success* report produced by the *National Mathematics Advisory Panel* (2008) expected teachers to have mathematical content knowledge beyond the level that they were assigned to teach. In addition, the report alluded to the necessity for future research "to create a sound basis for the preparation of elementary and middle school teachers within pre-service teacher education (p. xxii)."

Despite these recommendations, many teacher development programs did continue to design and offer courses in a traditional lecture-style structure rather than develop and implement them as recommended changes for the preparation of new mathematics teachers (Weber, 2004; Speer, Smith III, & Horvath, 2010; Weber, 2012). Further, little effort has been made to connect collegiate mathematics content courses with the school mathematics content expected to be taught by teachers in their future classes (Cuoco, 2001). These reports underlined the importance of the design of programs that support teachers to develop a *solid knowledge of mathematics* (CBMS, 2001). However, the interpretation of what constituted a solid knowledge of mathematics by each teacher educator was quite different. For many teacher educators, taking advanced mathematics courses and getting satisfactory grades from these courses were considered as the means to develop this knowledge. This kind of misinterpretation of solid knowledge of mathematics was perceived as a challenge for the qualification of prospective mathematics teachers (CMBS, 2001).

Considering the status of teaching, the high attrition rates of mathematics teachers during the first years of teaching, and the public belief about low level teacher quality in terms of mathematical content, it was clear that the challenge had not yet been overcome through the teacher education programs and mathematics content courses. According to the data from the 2011 public attitude poll (Bushaw & Lopez, 2011), 49% of the participants selected the internet as a means for the high-quality instruction rather than teachers. The *Mathematics for the 21^st^ Century* international study (Schmidt et al., 2007) comparing mathematics teachers' readiness to teach and their content knowledge also revealed findings confirming the public's dissatisfaction concerning teachers. The authors in this study posited the main difference among United States, Taiwanese, and Korean mathematics teachers might be the pedagogical preparation gap between teacher education models of the countries. This discrepancy could be an influential factor in teachers' level of understanding of mathematical concepts needed for teaching. These findings give evidence towards the importance of quality teacher education models and their ability to prepare mathematics teachers with the required skills and knowledge.

For many years, teacher education programs adopted a transmission model, which prioritized content knowledge acquisition as the only requirement for teachers to teach

(Darling-Hammond, 2006). In other words, in this model of teacher education, anyone knowing mathematics very well would be eligible to teach mathematics in schools. Findings from research (Monk, 1994; Rivkin, Hanushek, & Kain, 2005; Ball, Hill, & Bass, 2005) in the last two decades on the characteristics of mathematics teachers and their link to students' achievement demonstrate knowing mathematics is not sufficient for teachers to teach mathematics effectively.

In light of these developments, teacher education programs began to emphasize gains in pedagogical knowledge in addition to mathematical content knowledge. However, advanced mathematics courses still dominated pre-service programs and coursework. In addition, though deep understanding of mathematical concepts has been emphasized throughout reports since 2000, teacher preparation programs mainly interpreted it from what I term, a perspective of *mathematical pedagogy*. According to this perspective, content courses provide prospective teachers an opportunity to acquire advanced level subject-matter knowledge, which might be difficult to unpack during teaching, while mathematical methods courses are designed to emphasize general educational theories and develop PSTs' instructional strategies specifically for mathematics. In contrast, a *pedagogical mathematics* perspective envisions the interpretation of mathematics content specifically for the teaching profession. In this perspective, content courses would be designed to support teachers to develop subject-matter knowledge specific to teaching in school, while mathematical methods courses would keep the same purpose of equipping prospective teachers with pedagogical content knowledge. The latter approach to mathematics teacher preparation was recently recommended in the *Mathematical Education for Teachers II* report (CBMS, 2012).

The *MET II* report (CBMS, 2012) indicated the same consideration with its coursework recommendations for middle school teachers as the previous reports. The document specified advanced mathematics courses for prospective middle school teachers to take for their professional preparation. These recommendations were asserted as an alignment of teacher education programs for the CCSSM, and stressed the experience of advanced mathematics content through reasoning and sense making, so that prospective teachers could develop deep understanding of this content in relation to both elementary and high school mathematics concepts.

The *MET II* report (CBMS, 2012) recommended prospective middle school teachers to take 9 credits of advance mathematics courses such as calculus, discrete mathematics, and number theory; and at least 15 credits of mathematics courses specific to the middle school mathematics curriculum and offered only for middle school mathematics teachers. These mathematics courses revisited middle school concepts in number and operations, geometry and measurement, algebra and number theory, and statistics and probability, with an emphasis on deep understanding. In this interpretation of deep understanding, prospective teachers know more than simply how to “do” middle school mathematics, and now also develop a special kind of mathematical content knowledge, germane specifically to the work of teaching mathematics, which is referred to by Ball and her colleagues (2001) as *Specialized Content Knowledge* (SCK). *SCK* is a knowledge component of the *Mathematical Knowledge for Teaching* (MKT) framework (Ball, Lubinski, & Mewborn, 2001; Ball, 2003; Ball, Thames, & Phelps, 2008).

Mathematical Content Knowledge

The *MKT* framework elaborates *Pedagogical Content Knowledge* (PCK) (Shulman, 1986) for the domain of mathematics, and extends this conceptualization for subject-matter knowledge. The framework has two separate main domains: 1) Subject-Matter Knowledge, and 2) Pedagogical Content Knowledge. According to the *MKT* framework, mathematics teachers need to acquire and develop general mathematical content knowledge and a specialized one for teaching: *SCK*. *SCK* was defined as special mathematical content knowledge, which bridges well-known mathematical procedures with where they come from and why they were used in a certain way. With *SCK*, mathematics teachers could analyze the mathematics behind students' mistakes, hypotheses, propositions, and solutions for problems. Moreover, the efforts for teachers to develop *SCK* through teacher education could create opportunities for them to integrate advanced and school-based mathematical content knowledge (Moreira & David, 2008).

The *MKT* framework was developed to specify qualifications for the work of mathematics teaching and advocated that knowing formalized mathematical content on its own would not be sufficient for teachers. The conceptualization of *SCK* within the *MKT* framework served as evidence for the necessity of teacher education (Hill, Sleep, Lewis & Ball, 2007) and the importance of building knowledge for changes projected by reform-based standards (Goertz, 2010). This conceptualization also increased the challenge concerning how the development of such knowledge through teacher education programs would be accomplished.

As mentioned, organizations such as the *Association of Mathematics Teacher Educators* (AMTE) and CBMS had recently made recommendations to mathematics departments. These recommendations included collaboration with teacher education programs in light of the *MKT* framework, as well as laying out possible experiential opportunities for the development of teachers' mathematical content knowledge. Although teacher education programs have been theorized to be crucial in facilitating experiences, which support teachers' knowledge development, the *Foundations for Success* report (NMAP, 2008) stated that there were few studies addressing the impact of teacher education programs on teacher knowledge development. In regard to the information found in the *Foundations for Success* report, it can be concluded that there is a research need to investigate the qualities and aspects of teacher education programs in terms of the development of teachers' *SCK*, which enables them to approach mathematical tasks and representations from a conceptual as well as procedural standpoint.

Technology as a Catalyst

One possible way to accelerate teachers' development of mathematical content knowledge, emphasizing conceptual understanding and procedural proficiency, is the use of electronic technologies. Research has demonstrated that effective use of technology supports students' development of conceptual understanding (Mann, Shakeshaft, Becker, & Kottkamp, 1998; McCoy, 1996; Wiske, Franz & Breit, 2005; Roschelle, Shechtman, &

Tatar, 2010). The same premise may hold true for the development of teachers' *SCK* when technology is used effectively as a tool to construct required mathematical content knowledge.

CBMS (2001; 2012), with *MET I* and *II* reports, recommended that mathematics teachers develop an understanding of mathematical concepts while using technology within undergraduate mathematics courses. The type of technology recommended for teachers to have experience with during their education varied from programming-based technologies such as C++ to dynamic geometry software. Experiences with such technologies were claimed to support teachers' experimentation with mathematics and deep understanding with different representations of complex mathematical concepts (CBMS, 2012). Using technology in order to assist teachers in becoming more knowledgeable in mathematics teaching would also help to address the technology principle of *PSSM* (NCTM, 2000). Through learning with technology, teachers would gain a sensibility about how to more effectively use technology in their teaching.

Such expectations for teachers to use technology effectively in instruction pushed scholars in the field of education to revisit the theoretical framework for teacher knowledge to explicitly address technology. As a result, a framework called *Technological Pedagogical Content Knowledge* (TPACK) emerged (Koehler & Mishra, 2005).

TPACK is composed of 1) the knowledge of teaching content with technology, 2) the knowledge of instructional decisions and representations for teaching content with technology, and 3) the knowledge of students' learning with technology (Niess, 2008). Grandgenett (2008) recommended that teacher education programs integrate technology, content, and pedagogy within their coursework in order to prepare teachers to gain a disposition to experiment with new technologies. He also recommended offering teaching methods and mathematics content courses that support the development of *TPACK*.

According to the *TPACK* framework, mathematical content knowledge is evolving due to the infusion of technology. Grandgenett (2008) gave fractal geometry as an example of how school mathematics has been expanded with the use of technology. This new evolution of knowledge resulting from interacting with technology was termed *Technological Content Knowledge* (TCK) by Koehler and Mishra (2005). Regarding *TCK,* prospective mathematics teachers are expected to develop mathematical content knowledge as a result of interactions with the content through technology.

For teachers to develop *TCK,* experience with technology is an indisputable requirement within their teacher education programs. However, there is still a need for further research, which investigates experiences that benefit or limit the development of *TCK*. To this date, teaching methods courses have been utilized as sites to aid the development of *TCK* rather than mathematical content courses (Niess, 2008). Also, studies exploring *TCK* throughout teacher education programs do not show the influence of technology on their content knowledge, but focused instead on teachers' beliefs about this construct. Bowers and Stephens (2011) also stated that there is a lack of research on the development and nature of *TCK*.

Inserting Beliefs into the Equation

Although technological knowledge and *TCK* are necessary, they are not sufficient if teachers do not also feel confident using these knowledge to facilitate student learning (Ertmer & Ottenbreit-Leftwich, 2010). This point seems to be particularly true for novice teachers who question the effectiveness of using technology for learning gains. For other teachers, the image of technology integration might seem scary and intimidating because of the perception that technology can replace teachers by providing an easier way to learn and construct knowledge (Dawes, 2001).

The development of knowledge also depends on teachers' beliefs about mathematics as a discipline. Fuson, Kalchman and Bransford (2005) shared two different teachers' preconceptions about mathematics. According to one preconception, teachers might view mathematics as a discipline solely comprised of computations and rules to find the correct answer. They might consider that doing mathematics is an innate ability only bestowed on some people. Teachers might also preconceive mathematics as a discipline for problem solving and sense making. In this second preconception, mathematics is evolving with the invention of new mathematical procedures depending on time and place (Fuson et al., 2005). The variety of combination of beliefs teachers hold regarding mathematics and technology will certainly mediate teachers' development of *SCK*. While exploring the impact of technology and related teacher education experience for the development of teachers' mathematical content knowledge, it seems necessary to investigate pre-service mathematics teachers' beliefs about mathematics, teaching mathematics, and teaching mathematics with technology. Without inserting these factors into the equation, it might be difficult to lay out the big picture for the phenomenon under investigation.

Focus of the Study and Research Questions

Assuming that teachers need to develop and acquire *SCK*, teacher education programs should seek to offer mathematics courses that facilitate the development of such knowledge. Researchers also need to investigate the effectiveness of these courses as well as which experiences and factors enhance or restrain teacher knowledge development (Hill et al., 2007). "Teachers' opportunities to learn can help them develop their own knowledge about mathematics" (Kilpatrick, Swafford, & Findell, 2001, p. 429). My personal experience with technology led me to hypothesize that technological tools enable teachers to develop their own mathematical content knowledge.

Computer tools create opportunities for users to interconnect mathematical topics in a dynamic and interactive way. These tools make the exploration of real life phenomena possible, allow learners to be exposed to central ideas, and create new mathematics to learn (Cuoco, Benson, Kerins, Sword, & Waterman, 2010; Fey, Hollenbeck, & Wray, 2010). The use of virtual manipulatives provides teachers an opportunity to experiment with geometry. As an example, consider an objective of developing conceptual understanding for a triangle's area formula. Interaction with technology in this scenario would enable users to question the validity of the theorem for different conditions (Hollenbeck, Wray, & Fey, 2010). In Euclidean geometry, the area of the largest triangle inside a given rectangle is half of the area of the rectangle. Such theorems can be discovered through paper and pencil, but the use of dynamic geometry software would enable a more time-efficient discovery in a systematic way. Furthermore, the dynamic functionality of this software would allow users to observe how the area of a triangle is related to the area of a rectangle, and in which cases the area of the triangle is the largest and why.

In regard to the affordances of dynamic geometry software on the construction of geometry content knowledge, I examined the beliefs and experiences of pre-service mathematics teachers and how these beliefs and experiences impact their content knowledge development process within a graduate geometry course. Through this geometry course, dynamic geometry software was utilized as a cognitive tool. I assumed that technology would influence 1) the nature of common content knowledge of mathematics (Ball et al., 2008), and 2) the development of mathematical content knowledge specific to teaching. This study tested the extent of these assumptions.

My research findings from this study inform teacher educators regarding the preparation of mathematics teachers and create a foundation to restructure teacher education programs and mathematics courses around these experiences. The conceptualization of technology, mathematical content knowledge and beliefs, and the influence of technology for both the nature and development of these knowledge and beliefs are elaborated in the next chapter. With respect to the rationale I laid out in this chapter and theoretical framework provided in the next chapter, the following research questions guided my study:

1. How does a Technology Integrated Geometry Course influence pre-service mathematics teachers' development of *SCK*?

2. How does a Technology Integrated Geometry Course influence pre-service mathematics teachers' development of *TCK*?
3. How are pre-service mathematics teachers' beliefs related to their SCK development?
    a. How are pre-service mathematics teachers' beliefs about *the discipline of mathematics* related to their SCK development?
    b. How are pre-service mathematics teachers' beliefs about *teaching mathematics* related to their SCK development?
    c. How are pre-service mathematics teachers' beliefs about *technology* related to their SCK development?

CHAPTER TWO – REVIEW OF THE LITERATURE

This chapter focuses on the review of the literature related to the research questions of the study, and elaborates on the theories and concepts tied to the research. More specifically, I present theories and findings under three main topics: 1) Teacher knowledge as a construct, 2) Teacher beliefs as a construct, and 3) Instructional technology and knowledge. Each of these topics is addressed and described as literature bases and theoretical frameworks within my three manuscripts:

Topics	**Manuscript Covering the Topic**
Teacher knowledge as a construct	1st Manuscript
Teacher beliefs as a construct	3rd Manuscript
Instructional technology and knowledge	2nd Manuscript

Table 2.1: Theoretical Frameworks and their Connection to the Manuscripts

The first section addresses knowledge as a cognitive construct and how it has been viewed and defined in mathematics education research. This section focuses on teacher knowledge. It specifically spotlights *Pedagogical Content Knowledge* (PCK) (Shulman, 1986) and *Mathematical Knowledge for Teaching* (MKT) (Ball, Thames, & Phelps, 2008), their relationship to each other, along with recent findings for the development of these knowledge bases in the presence or absence of technology during instruction. In the second section, I define, describe, and discuss teacher beliefs as a construct and present related findings from recent studies investigating mathematics teachers' beliefs. The final section presents studies that define and categorize instructional technology, as well as the knowledge necessary for mathematics teachers to utilize technology effectively during the instruction. Particularly, I emphasize

Technological Pedagogical Content Knowledge (TPACK) (Koehler & Mishra, 2005) and the development of this knowledge by mathematics teachers. Following these sections I present my theoretical frame for the study, explicating how I view each construct and how I utilize them in my data analysis.

Knowledge Bases for Teaching Mathematics

Defining Knowledge as a Construct

Knowledge as a cognitive construct is one of the terms scholars have struggled to clearly define and describe. It is difficult to determine what to label as knowledge and how it is different from other cognitive constructs such as beliefs.

Verloop, Driel and Meijer (2001) defined knowledge "as an overarching, inclusive concept, summarizing a large variety of cognitions, from conscious and well-balanced opinions to unconscious and un-reflected intuitions" (p. 6). Regarding this definition, one would accept that knowledge is not only the result of the accumulation of information whose correctness was confirmed in our minds. To elaborate on its definition, Lehrer (1990) further pointed out the multiplicity of knowledge by looking at the word of *knowing*. In this respect, knowledge can represent competence, acquaintance, or information. We might know how to play a piano (competence), recognize our relatives' faces and call their names (acquaintance), or know that water evaporates at 100° C (information). Lehrer (1990) also stated that knowledge in the form of competence or acquaintance can be encoded in the information form of knowledge. As an example, a person needs to know musical terms and notations (information form of knowledge) to be able to play a piano (competence form of knowledge).

Lehrer (1990) emphasized the correctness of the information for it to be labeled as knowledge. In other words, receiving information from a resource does not guarantee that it would be called knowledge. Regarding the last statement, one would ask how we might judge the correctness of information. Lemos (2007) laid out the discussion of whether information is correct or not in terms of its correspondence to the negotiated facts. According to Lemos (2007), there are three types of knowledge: 1) *how to* knowledge (similar to Lehrer's competence form of knowledge), 2) acquaintance knowledge, and 3) propositional knowledge (similar to Lehrer's information form of knowledge), which is the knowledge of facts and true propositions. A proposition can be called true if and only if it corresponds to the facts. For example, the proposition of "four times two is equal to six" is a false proposition given by a pre-school child. The child might think that the statement is true, but it does not show that s/he knows the multiplication operation accurately because the statement does not correspond to the facts about multiplication. Whether a proposition is true or corresponding to the facts also determines the differentiation of knowledge from beliefs. For example, a person might believe a proposition is true. This person's belief can be labeled as knowledge if and only if the proposition depends on the facts. If the truth of the proposition cannot be proven by the facts, then the proposition only represents the person's belief.

Lemos's (2007) knowledge categorization and his differentiation of knowledge from beliefs can similarly be mentioned for teacher knowledge. While propositional knowledge for teaching generates theoretical teacher knowledge, *how-to* knowledge would create teachers' practical knowledge coming from years of experience (Verloop, Driel & Meijer, 2001). Teachers gain insights about teaching and pedagogical theories through teacher education or professional development programs. These theories construct teachers' propositional knowledge consisting of pedagogical and subject-matter related facts. Teachers also create practical knowledge about teaching through teacher education programs and teaching practice (Fenstermacher, 1994).

Up until the 1980s, educational research was conducted to determine a practical knowledge base for teaching by looking at effective teaching practices of expert teachers in classrooms. However, this approach for the foundation of teacher knowledge has been criticized because teacher behavior is not the only determinant of teacher knowledge. In addition, the characteristics of a context embedded within an effective teaching episode limit its applicability for other teachers and their classrooms. These criticisms enabled researchers to look for a more theoretical knowledge base for teaching. Fenstermacher (1994) labeled this theoretical knowledge base as formal knowledge for teaching, which differs from practical knowledge. When comparing formal and practical knowledge, he pointed out that teachers are mostly accountable for the production of practical knowledge, while researchers and scholars take on the responsibility for the production of formal knowledge. Furthermore, he cited the difficulty in differentiating knowledge from beliefs. According to Fenstermacher (1994), epistemic merit, which depends on warrant and evidence, is the most important component of knowledge that differentiates it from beliefs. As an example of this differentiation, a teacher could *believe* in the correctness of information or an event, or could *know* it through empirical observation or deductive reasoning. While Fenstermacher (1994) differentiated teacher knowledge from beliefs, Verloop, Drier, and Meijer (2001) did not view knowledge as a separate construct from beliefs. They defined teacher knowledge as intertwined with beliefs, conceptions, and intuitions in teachers' minds. However, I consider beliefs are related to values, attitudes and opinions, while knowledge consists of facts and true propositions (Pajares, 1992).

Schoenfeld (1983) approached knowledge as the *resource* component of pure cognition, and considered it to be in interaction with and influenced by other components such as control behavior (metacognition) and beliefs. Though he viewed knowledge holistically, his view still emphasizes that the formal knowledge produced by scholars consists of procedural and factual knowledge that might be used as a resource when needed. Being holistic or separate from other cognitive processes, teacher knowledge is crucial in informing teachers' plans, decisions, and practices.

In light of the literature defining knowledge above, I define knowledge for this study as cognitive products, which consist of procedural and conceptual propositions that might be projected onto facts negotiated by others as valid. While the certainty and validity of facts could be judged easily for mathematical knowledge, it might not be straightforward for pedagogical premises because pedagogical premises might be context dependent and open to interpretation. With this definition in mind, I next examine the role of knowledge with respect to teaching. In the following section, I share how knowledge was defined specifically for the profession of teaching, and how these knowledge definitions differ in varied teacher knowledge models that were used in and framed teacher education programs and professional development activities.

Knowledge for Teaching

Though there have been efforts to reveal personal teacher knowledge through investigating teacher classroom behaviors and examining the propositions and facts behind their actions (Nespor & Barylske, 1991; Clandinin & Connely, 1996), research during the second half of the 1980s generated and described several new models and shared components of teacher knowledge (Leinhardt & Smith, 1985; Shulman, 1986). Leinhardt and Smith (1985) demonstrated teacher knowledge for mathematics as composed of subject-matter knowledge and lesson-structure knowledge. The authors defined subject-matter knowledge as the knowledge of "concepts, algorithmic operations, and connections among algorithmic procedures, the subset of the number system being drawn, the understanding of classes of student errors, and curriculum presentation" (p. 247), and described lesson-structure knowledge as composed of skills to plan and run an instruction smoothly. From these definitions, Leinhardt and Smith (1985) considered knowledge related to curriculum as a subset of subject-matter knowledge. Within this model, both subject-matter and lesson-structure knowledge are viewed as intertwined in such a way that the existence of one knowledge type would influence the behavioral enactment of the other knowledge type.

Fennema and Franke (1992) found two limitations in Leinhardt and Smith's (1985) teacher knowledge model in terms of its applicability in mathematics education. The authors critiqued that mathematics studied as a component of teacher subject-matter knowledge emphasizes procedure and algorithm acquisition rather than understanding. They stated that the description of subject-matter knowledge in this way only causes teachers to view mathematics as a discipline consisting of small procedural mathematical skills and ideas, but not as a holistic subject. In other words, regarding this knowledge model, teachers develop their subject-matter knowledge made up of small ideas, but could not see the big picture within teaching and learning mathematics. Secondly, teacher knowledge lacks attention on individual students' learning of mathematics in Leinhardt and Smith's model (1985). The authors paid attention to the teachers' pre-developed scripts, routines and agendas, which were described as skills needed for lesson-structure knowledge development. However, the emphasis on pre-prepared routines for teaching does not allow teachers to utilize students' individual learning strategies in order to increase instructional effectiveness.

Shulman's (1986) model includes three kinds of content knowledge: 1) content knowledge, 2) pedagogical content knowledge (*PCK)*, and 3) curricular knowledge. According to his description, content knowledge is composed of facts, concepts, and understanding of structures within a given subject. Shulman (1986) described *PCK* as special knowledge, which helps teachers transfer what they knew as subject-matter to their instruction in a form that facilitated students' comprehension. *PCK* of a subject includes knowledge of multiple representations, analogies, explanations and examples related to the subject, an understanding of the subject's characteristics in terms of how they make the instruction difficult or easy, and comprehension of possible misconceptions in order to enable students' learning.

Shulman (1986) defined curricular knowledge as knowledge which "underlies the teacher's ability to relate the content of a given course or lesson to topics or issues being discussed simultaneously in other classes." (p. 10). Via curricular knowledge, teachers choose alternative materials for the instruction, and relate a subject or concept to previously covered subjects or concepts. With the emergence of *PCK* as a construct in the field of education, Shulman contributed to the professionalization of teaching, and helped clarify what makes a teacher different from someone else who is knowledgeable in that same subject.

Leinhardt and Smith (1985) described subject-matter knowledge as not only consisting of facts, concepts, procedures, and algorithms for a specific subject, but also with its link to possible student errors and curriculum interpretation. In this respect, even though they did not identify or label their knowledge components as Shulman (1986) did, it is clear from their definition of subject-matter knowledge that the authors were aware of the existence of special knowledge components under subject-matter knowledge. Shulman (1986) labeled the aspect of Leinhardt and Smith's (1985) subject-matter knowledge that emphasizes teachers' understanding of student errors as *PCK*, and elaborated upon this new term. Another difference between Shulman's (1986) and Leinhardt and Smith's (1985) models of teacher knowledge concerns understanding of curriculum. While Leinhardt and Smith (1985) considered curriculum as a subset of subject-matter knowledge in their model, Shulman (1986) identified it as a new set of knowledge that is mutually exclusive from the content and pedagogical knowledge. Shulman (1986) separated out curriculum knowledge from content and pedagogical content knowledge. However, it does not mean that there is no relationship between content knowledge and curriculum knowledge. Enhancement in one knowledge type could still support the development of other knowledge type while they are demonstrated in mutually exclusive ways.

Content knowledge in Shulman's (1986) teacher knowledge model was defined differently for different subjects. For example, it was defined for mathematics as conceptual understanding of mathematical knowledge taught during teacher education of mathematics teachers (Baumert et al., 2010). Studies in mathematics education (Even, 1993; Krauss et al., 2008; Baumert et al., 2010) examining the differentiation of content knowledge from *PCK* and looking at their interaction were one of the important research efforts in the field of education.

After the introduction of *PCK* into the educational lexicon in 1986, several educational researchers in and outside of mathematics education investigated this construct within the classroom context and created new models of teacher knowledge (Carpenter, Fennema, Peterson, & Carey, 1988; Howey & Grossman, 1989; Grossman, 1990; Even, 1993). For example, Grossman (1990) described teacher knowledge for teaching English in terms of four general areas: 1) subject-matter knowledge, which was defined as knowledge of facts, concepts, relationships among concepts and facts, and the structure of the subject-matter, 2) general pedagogical knowledge, which was defined as knowledge and beliefs concerning learners, learning, instruction, classroom management, and aims of education, 3) *PCK*, which was composed of knowledge and beliefs about the purposes of education, students' ways of learning, curricular knowledge, and instructional strategies pertaining to a specific subject-matter, and 4) knowledge of context, which was defined as the understanding of specific contexts in order to apply teacher knowledge to school settings and individual student differences.

A similar teacher model to Grossman's model was given by Fennema and Franke (1992), who discussed mathematics teacher knowledge consisting of four facets: 1) knowledge of mathematics, 2) knowledge of mathematical representations, 3) knowledge of students, and 4) general knowledge of teaching and decision making. Fennema and Franke (1992) stated the necessity of making sense of the nature of mathematics and its mental organization for teachers' acquisition of the knowledge of mathematics. They defined the knowledge of mathematical representations as the translation of complex subject-matter knowledge into representations so that students could make sense of what is presented.

The first two components of teacher knowledge by Fennema and Franke (1992) indicate that there is not only one mathematics or mathematical knowledge teachers need to acquire or develop for instruction; teachers are expected to be knowledgeable in mathematical concepts and procedures as well as in decomposing that knowledge into a form for students' learning. Fennema and Franke (1992) labeled the knowledge of how students learn and acquire mathematical knowledge, and how it might guide instructional decisions as the knowledge of students. Regarding their descriptions, the second and third components together in Fennema and Franke's (1992) model partially corresponds to Shulman's (1986) *PCK*. Shulman also described *PCK* as knowledge of particular mathematical representations for students' understanding and of students' learning strategies.

Fennema and Franke (1992) described the last component, general knowledge of teaching and decision making as knowledge and skills of planning, implementation, evaluation and reflection of the instruction. Fennema and Franke (1992) modeled teacher knowledge as a developmental process, as teacher knowledge changes as a result of teachers' evaluation of and reflection on their experiences in and outside the classroom. This process is mediated by interactions among a teacher's knowledge of mathematics, mathematical representations, students' learning and teaching/decision making embedded within the specific context of the teacher's classroom.

Mathematical Knowledge for Teaching

While *PCK* is a useful term in investigating the relationship between content and pedagogical knowledge for the teaching profession, it could not answer all questions pertaining to the complexity of teaching mathematics. Ball and Bass (2000) pointed out to overcome this complexity by bridging content and pedagogy more specifically for teaching mathematics. They also stated the necessity of exploring *PCK* more in-depth within teaching practices.

Ball, Lubienski and Mewborn (2001) criticized the focus of research on mathematics knowledge and its foundation from the examination of mathematics curricula. Rather than a traditional approach of looking at the content to configure mathematical knowledge needed for teaching, they claimed that classrooms should be the research sites in order to reveal of the type of knowledge needed by mathematics teachers. Hiebert, Gallimore, and Stigler (2002) raised a similar criticism; while teacher educators in research institutes advocated that knowledge gained from research could improve the perception of teaching as a profession, the authors claimed that practitioner knowledge could be more reliable for the same purpose. These criticisms and the shift of emphasis in the field of education from formal to practical knowledge allowed Ball and her colleagues (Ball, Lubienski, & Mewborn, 2001; Ball, 2003; Ball, Thames, & Phelps, 2008) to develop and introduce the construct of *Mathematical Knowledge for Teaching* (MKT).

According to the *MKT* framework, mathematics knowledge for teaching is initially separated into subject-matter knowledge and pedagogical content knowledge. These two facets are then further subdivided into three knowledge components. The following diagram presents each of these components of the *MKT* Framework (Ball, Thames, Phelps, 2008). I begin with an unpacking of two constructs under subject-matter knowledge. I do not describe horizon content knowledge under subject-matter knowledge and pedagogical content knowledge in this diagram because it is not the focus of this study.

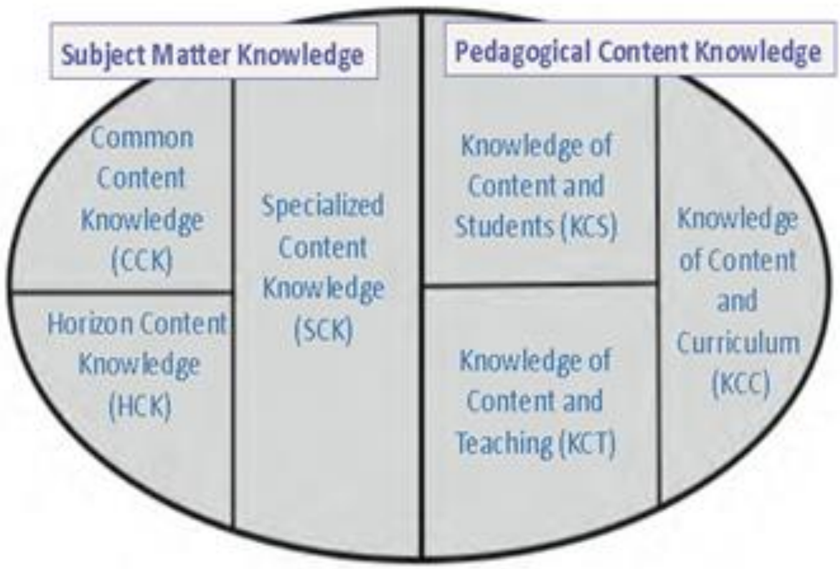

Figure 2.1: Representation of the MKT Framework (Ball, Thames, Phelps, 2008)

Common Content Knowledge (CCK) is the "knowledge of mathematics that was common across professions and available in the public domain" (Hill, Sleep, Lewis, & Ball, 2007, p. 131). The availability of CCK enables teachers to "compute, make correct mathematical statements … , solve problems" (Hill, Schilling, & Ball, 2004, p. 16), "know the material they teach … , recognize when their students give wrong answer or when the textbook gives an inaccurate definition …, use terms and notations correctly" (Ball, Thames, & Phelps, 2008, p. 399). However, the authors who introduced this construct in the field of mathematics education also underlined that this mathematical knowledge is common to any profession which uses/applies mathematics.

Regarding these definitions from different articles, for my study I define CCK as mathematical knowledge that an undergraduate student, who is not majoring in mathematics education, might develop through his/her tertiary study. CCK is factual, conceptual, procedural and algorithmic knowledge which enables teachers to recognize mathematical facts, procedures, strategies, to define concepts correctly with mathematically acceptable terms and notations, and to differentiate the correct answer from the incorrect ones for a given problem. For example, a secondary school mathematics teacher knows that $(x - h)^2 + (y - k)^2 = r^2$ is the algebraic representation for any circle where r denotes the length of its radius and (h, k) denotes its center. In addition, the teacher also recognizes that a circle on the Cartesian coordinate plane is not a function by applying the vertical line test. Both of these examples are a part of the teacher's CCK because the fact about the circle's algebraic representation and the procedure to determine whether a given shape is a function or not can be acquired, developed, and conceived by people in other professions as well.

Specialized Content Knowledge (SCK) is the knowledge of mathematics exclusively needed for the teaching profession (Ball, Hill, & Bass, 2005), which includes "building and examining alternative representations, providing representations, and evaluating unconventional student methods" (Hill, Schilling, & Ball, 2004, p. 17). The following list of teacher skills describes what constitutes SCK and how it is different from CCK:

- Showing and representing mathematical terms and operations visually,
- Providing mathematical reasons for common procedures,
- Understanding mathematics behind students' unusual procedures and generalizing them if needed,
- Constructing real life problems related to specific mathematical concepts,

- Examining and unraveling the source of students' mathematical errors (Ball, Hill, & Bass, 2005; Hill, Sleep, Lewis, & Ball, 2007; Ball, Thames, & Phelps, 2008)

One of the main characteristics of SCK which differentiates it from CCK is it is the knowledge used for contributing to students' learning, but not taught directly to students. That said, a teacher does need to have and develop SCK, a special kind of subject-matter knowledge, not in order to teach to students the same as CCK, but to utilize it when needed. In this respect, I consider that SCK is secondary to students' learning of mathematics during instruction. The teacher focuses on decomposing his/her CCK to achieve content-related learning goals, but utilizes his/her SCK to be able to overcome possible problems with respect to students' learning when needed. For example, a student might ask why s/he is using vertical line test to identify a given graph is a function or not. The answer to this question is not in the scope of the instruction if the teacher has not included it within the learning goals. In other words, learning the reasoning behind the vertical line test might not be planned as a learning goal. However, a student might still ask such a question; and availability of SCK for the teacher about this concept helps him/her strengthen the student's understanding.

Regarding this example, conceptual knowledge can be a part of the teacher's SCK as long as s/he does not aim or plan to teach it to students. One might wonder whether conceptual knowledge should be categorized as CCK or SCK. Ball and her colleagues raised the same concern (Ball, Thames, & Phelps, 2008):

> Some might wonder whether this decompressed knowledge is equivalent to conceptual understanding. They might ask whether we would not want all learners

> to understand content in such ways. Our answer is no. What we are describing is more than a solid grasp of the material. (p. 400)

Ball, Thames and Phelps (2008) considered that students might need to learn each detail for subject-matter knowledge that would enable them to understand the content conceptually. To expand the discussion about labeling conceptual knowledge as CCK or SCK, the authors gave an example:

> The mathematical demands of teaching require specialized mathematical knowledge not needed in other settings. Accountants have to calculate and reconcile numbers and engineers have to mathematically model properties of materials, but neither group needs to explain why, when you multiply by 10, you "add a zero." (p. 401)

Regarding my example, and the example given by Ball and her colleagues above, SCK includes conceptual understanding, but it is not equal to that. It is more than that. In Ball and her colleagues' example, teachers' knowledge of why they "add a zero" while multiplying a whole number by ten is labeled as SCK. It is a kind of trick for students' computational fluency. The vertical line test in my example is another trick teachers would use for computational fluency. However, a student might still ask why it does always work, and what mathematics is behind this trick. A correct response to this student's why question would be the result of the availability of the teacher's SCK. The teacher needs to decompose his/her conceptual understanding about functions to answer this question.

Second, I still do classify CCK as knowledge that teachers are planning to teach, and SCK as knowledge they are not planning to teach. SCK is all about spontaneous and coincidental moments within the classroom during which the teacher needs to understand mathematics within situations, and decompose his/her complex and advanced conceptual knowledge so as to resolve mathematical problems. This problem might be to determine a response to a why question asked by a student about a procedure, and the teacher might need to think conceptually during that time to answer it. It is SCK because she might not have planned to teach it during that time.

Regarding the emphasis on SCK as mathematical knowledge not directly taught to students, I can conceptually compare and contrast CCK and SCK by looking at the difference between "academic mathematical knowledge and mathematical knowledge associated with the demands of school teaching practice" (Moreira & David, 2008, p. 24). Academic mathematical knowledge can be viewed as mathematical knowledge in which mathematical concepts are defined correctly and connected in a logical organization. The second type of mathematical knowledge is helpful in addressing issues occurring in classroom. In regard to the definitions of these two knowledge bases, CCK can correspond to academic mathematical knowledge; and SCK can correspond to mathematical knowledge associated with the demands of school teaching practice. Moreira and David (2008) found that acquiring academic mathematical knowledge does not always enable teachers to organize their mathematical content knowledge needed for classroom practice. In other words, one type of knowledge is not sufficient for the development of the other. In this context, academic mathematical knowledge does not enable teachers to develop mathematical content knowledge needed for classroom practice. Of course, the difference between academic mathematical knowledge and mathematical knowledge associated with the demands of school teaching practice is not the only means that explains the overall difference between CCK and SCK.

Prior to the emergence of the SCK concept in the literature, mathematical content knowledge special and unique to the profession of teaching had been identified and considered as necessary for teachers. For example, Leinhardt and Smith (1985) pointed out the need for the development of mathematical content knowledge unique for the teaching profession, but did not refer to this knowledge as SCK. Their study aimed to explore the influence of teacher expertise on their subject-matter knowledge. While they found that expert teachers in the study had deeper subject-matter knowledge, the quality of this knowledge differed among them as well. The major difference for these participating teachers was on the level of emphasis on conceptual understanding compared to transmitting procedures. The authors provided the following example from their findings to describe this difference. Some expert teachers maintained the equivalence of two fractions by only raising them (multiplying both numerator and denominator), and did not mention dividing both numerator and denominator would also accomplish this. Another example concerned the representation of fractions on a rectangle. Teachers used rectangle slicing to demonstrate the equivalence of two fractions.

To represent ½, a teacher might draw a rectangle, slice it into two equal pieces, and shade one. The teacher might represent the equivalence of ½ to 2/4 by slicing each part equally a second time. In addition to these pedagogical actions, the teacher is expected to know the following mathematical operation is embedded within the slicing of the rectangle: as the line doubles the number of pieces in the whole as well as the number of pieces being considered, "multiplication by 2/2 is equivalent to drawing a single line" (Leinhardt & Smith, 1985, p. 269). Knowing such mathematics for specific representations is a part of SCK rather than CCK because the teacher does not need to transmit this knowledge to students for their understanding, but should be aware of the mathematics behind the representation to be able to overcome students' possible incorrect inferences and misconceptions.

A similar identification of mathematical content knowledge specific to the teaching profession was presented by Even (1993), who investigated the interrelation between content knowledge and PCK, and discovered that several pre-service secondary mathematics teachers participating in the study did not have a modern concept of function. The author defines the modern concept of function as the appreciation of the arbitrary nature of functions and univalence. According to the arbitrary nature, functions do not necessarily require a regularity, which can be described by a specific algebraic expression or a particular graph in a certain shape. Univalence of functions means that there is only one image of each element of the domain. Even (1993) also stated that there is still a need for *why* univalence is necessary for the function's definition even if a teacher has developed modern concept of function. Having an answer to this why question might emerge if the teacher has related SCK for the concept of functions. For the development of such specific subject-matter knowledge, the author concluded with the necessity of learning environments that promote strong constructions of mathematical concepts.

The difference between CCK and SCK was empirically documented after these two constructs were emerged. Hill, Schilling and Ball (2004) designed and tested measures assessing elementary mathematics teachers' content knowledge. One of the research aims for their study was the assessment of whether there is a difference between CCK and SCK. Statistical findings from factor analysis suggested that CCK and SCK are related but not equivalent. In addition, the authors considered that, through teacher education and professional development programs, teachers might develop CCK, but not SCK; or vice versa.

In general, the *MKT* framework makes an attempt to elaborate and refine Shulman's (1986) knowledge bases of content knowledge and *PCK*. According to Hill, Ball and Schilling (2008), Shulman's description of content knowledge shows similarities to CCK. Regarding this similarity, one of the main contributions of the *MKT* framework to the teacher knowledge model by Shulman (1986) is the introduction of SCK. SCK is practice-dependent knowledge which could be acquired and developed by practicing teachers, but not necessarily by mathematicians. In addition, the *MKT* framework compartmentalizes *PCK* in terms of teaching, students and curriculum. The authors identified different kind of *PCK* with respect to teachers' instructional decisions, their understanding of students' mathematical thinking and curriculum. The *MKT* framework also envisions knowledge of curriculum in terms of subject-matter knowledge and *PCK* separately as HCK and KCC.

Recent Literature on Mathematical Knowledge for Teaching

The *MKT* framework has provided educational researchers a new way to examine the influence of practicing and prospective mathematics teachers' knowledge on their classroom practices (Izsák, 2009), the quality of instruction (Hill et al., 2008), and students' achievement (Hill, Rowan, & Ball, 2005). It also generated a new framework for studies examining the effectiveness of professional development programs on the quality of teaching and teacher development (Hill & Ball, 2004). While some researchers focused on the examination of the validity and reliability of *MKT* as a construct to measure teachers' knowledge (Hill & Ball, 2004; Hill, 2010), others qualitatively investigated how *MKT* enables teachers to develop required skills for classroom practices and to reform their instructions (Sullivan, Clarke, & Clarke, 2009; Izsák, 2009).

Even though researchers used *MKT* as the framework for their examination of teacher knowledge, their sub-component knowledge definitions and the labels they used for these sub-components are not always similar to each other. For example, Izsák (2009) described three knowledge elements of teachers for fraction multiplication: 1) numerical aspects of multiplication, which represents declarative statements about multiplication of fractions, 2) unit structures which identifies fractions as parts of a whole or parts of parts, 3) pedagogical purposes for the illustrations of the computation method of fraction multiplication. These descriptions of knowledge elements show similarities to the definitions of CCK, SCK and PCK, respectively. However, Izsák (2009) did not label the elements as such in his examination of two middle grade teachers' *MKT* in relation to fractions, fraction multiplication and representations. Izsák (2009) operationally defined “knowledge for teaching” as knowledge a teacher uses when responding to his/her students. From the case study of two teachers, the author found that participating teachers’ limited capacity to reason with fractions and different unit structures caused constraints on their ability to form pedagogical purposes for different representations of the fractions and fraction multiplication. This finding implies teachers’ limited CCK and SCK, with respect to fraction multiplication, also hinders the development and availability of their *PCK*, the knowledge of how to utilize students’ different representations for instructional purposes.

Similar to Izsák (2009), Sullivan, Clarke and Clarke (2009) researched practicing K-12 teachers’ development of sub-knowledge components of *MKT* while they were asked to describe the content of an illustrative task on fraction division and to create a lesson around it. Unlike Izsák (2009), Sullivan, Clarke and Clarke (2009) explicitly examined and identified knowledge elements as CCK and SCK. Using teachers’ written responses, the authors determined that some teachers were not able to identify the focus of the content within the illustrative task. The same teachers in the study did not have knowledge about different student strategies for fraction division and comparison tasks. The authors reasoned although teachers had sufficient CCK for the division of fractions, their limited availability of SCK resulted in teachers’ lack of ability to transform a mathematical task into a lesson. While this study has some insight about how subject-matter knowledge and *PCK* are linked, it does not give detail information about relationships occurring for sub-domains of the subject-matter knowledge.

Research conducted by Hill and her colleagues (Hill & Ball, 2004; Hill, 2010) were large-scale quantitative studies that aimed to examine factors affecting teachers' *MKT* and its development. For example, Hill and Ball (2004) looked at the characteristics of a professional development program to improve elementary mathematics' teachers' CCK and SCK. Within this professional development program, elementary teachers attended one to three weeks summer workshop that offered content courses on number and operations by mathematicians and mathematics educators. After this summer workshop, teachers were followed up during the school year for 80 hours, and received a stipend for their participation. Compared to other professional development programs, the authors observed that this program was more focused on mathematical content. The authors described SCK for this study as "teachers' ability to unpack mathematical ideas, explain procedures, choose and use representations, or appraise unfamiliar mathematical claims and solutions" (p. 335). The results indicated teachers attending to the professional development program improve their CCK and SCK. The more teachers have opportunities to analyze students' solutions and to learn about mathematical communication and representation, the more they develop CCK and SCK. However, the authors could not create a clear model for professional development activities that would foster this improvement. While the authors concluded that there is an effect of the professional development program on the improvement of content knowledge in general, they did not specifically discuss or present results about whether or how it contributes to teachers' SCK development.

A narrowing of focus to literature addressing pre-service mathematics teachers' development of SCK produced limited results. One reason for this limitation could be the difficulty in differentiating SCK from other types of subject-matter knowledge or from aspects of *PCK* (Speer & Wagner, 2009). This difficulty could also explain decisions to orient studies on *MKT* in general rather than SCK, specifically (e.g. Hill, 2010). However, my search did uncover three studies that examined pre-service teachers' SCK development (Swars, Hart, Smith, Smith, & Tolar, 2007; Bair & Rich, 2011; Morris, Hiebert, & Spitzer, 2009).

Morris and her colleagues (2009) investigated the development of SCK during teachers' pre-service education. In this study, SCK was defined as knowledge of mathematics unique to teaching math, and considered as necessary knowledge for teachers to develop skill in specifying and unpacking learning goals into sub-concepts. In this respect, the authors examined how pre-service elementary teachers unpack learning goals into sub-concepts for planning, evaluation, teaching and learning. The participants responded to mathematical tasks which explored their ability to anticipate an ideal student response, to evaluate an incorrect student response, to evaluate a student's correct work, and to analyze a classroom lesson. Their written responses were coded according to pre-determined categories for each task, and they were scored zero, one or two according to the level of understanding apparent in the response. As a result, PSTs managed to identify sub-concepts for a learning goal in supportive contexts, but could not apply this knowledge for planning, evaluation, teaching and learning. Supportive contexts were the ones in which PSTs solved the problem by themselves, or examined students' incorrect responses. However, participants could not identify the sub-concepts for the learning goal when the context was non-supportive and the sub-concepts were hidden within the learning goal. The study demonstrated evidence for the development of pre-service teachers' SCK, but at the same time, indicated their development was limited as teachers had difficulty in using SCK for instructional purposes. Teachers might have SCK, but its enactment during instruction might come with more teaching experience.

While Morris and her colleagues (2009) studied pre-service teachers SCK through clinical interviews, Bair and Rich (2011) examined the same phenomenon over the span of two mathematics content courses. The authors questioned why some teachers are better in unpacking their SCK while teaching than others. Bair and Rich (2011) defined SCK as unique knowledge of teaching mathematics with understanding, where understanding was defined as having a sense of mathematical concepts as connected and related within the underlying structure of mathematics. The domain of number theory was the focus of the study. A grounded theory approach allowed authors to initially create a framework with five levels and indicators. When this framework was compared and contrasted with the Ball and Bass' (2000) eight descriptors of SCK, the number of levels was reduced to four components with five levels of indicators. According to Bair and Rich (2011), the four main components of their SCK progression are 1) explaining their reasoning, 2) using multiple standard representations, 3) relationships among conceptually similar problems, 4) problem posing. For each component, five levels indicated the progression of SCK from level zero, which only represents the CCK usage, to level four, which shows deep and connected SCK. The authors provided examples for each component and level.

This exploratory study demonstrated that teachers who developed SCK organized their instructional activities with respect to simultaneous use of their SCK and *PCK* together. PSTs in the study were asked to pose follow up questions for students in the area of quadratic functions. They were expected to create similar problems that addressed the same learning goal. PSTs with lower SCK could only maintain the learning goal by posing follow-up problems that included only trivial changes to numbers. When they were asked to create similar problems with non-trivial numerical changes, these teachers could not maintain the level of difficulty. On the other hand, PSTs having deep and connected SCK constructed follow-up problems with non-trivial numerical changes while maintaining the difficulty level of the problems. Additionally, these PSTs changed the context for the follow up problems and even wrote extension tasks. Teachers who had strong SCK on a specific domain of mathematics would integrate this knowledge with *PCK* while they were teaching. In addition, the study indicated PSTs who have insufficient CCK might still show development of SCK, but could not move to higher levels of SCK understanding.

Framework for Specialized Content Knowledge

Regarding knowledge definitions by Lehrer (1990) and Lemos (2007), an individual's knowledge is composed of true propositions supported by mathematical facts in which the individual has no doubt as to its certainty. The teacher might exhibit this knowledge through his/her speech or by writing his/her thought process to solve a mathematical problem. Either through speech or as a written artifact, mathematical facts or propositions stated by the teacher indicate the teacher's knowledge. While statements that include certainty appearing with academically approved mathematical facts represent teacher knowledge, others that involve any doubt and are stated without any justification would be categorized as teacher belief.

In using *MKT* to frame my study, I also need to clearly differentiate SCK from other forms of subject-matter knowledge or *PCK*. I categorize knowledge statements as evidence of CCK if the statement includes mathematical propositions, facts, concepts, procedures and their connections as long as they are tied to specific instructional goals. For example, suppose a learning goal for a lesson on geometry for ninth grade students is the identification and application of the Pythagorean Theorem, then all true propositions and facts that are required by the students to reach this goal would be classified as a part of a teacher's CCK.

While CCK includes true mathematical statements related to a defined learning goal, SCK might include the same statements, but more than that as well. A knowledge statement can be categorized as SCK if it demonstrates mathematical facts behind representations, unusual student procedures, and student errors. If a teacher is not required to know a mathematical proposition or reasoning, but somehow his/her previous experiences enabled him/her to develop this special mathematical knowledge which includes proposition or reasoning, and used it when an instructional situation necessitated its unpacking to help students' understanding or to meet their curiosity, then I would call this knowledge statement SCK rather than CCK. For example, a ninth grade student might ask the teacher why the Pythagorean Theorem works for any right triangle, which might prompt the teacher to share one or more of the mathematical proofs for this theorem. The teacher may not have planned to discuss or present this knowledge to students as a part of the lesson's learning goal, but responding to the student's curiosity allowed the teacher to demonstrate his/her SCK. While knowledge of various proofs of the Pythagorean Theorem may be an example of SCK for this specific lesson and grade, this knowledge might be categorized as CCK for a higher grade level. The task of identifying examples of SCK is therefore dependent on the mathematical level of the teacher and of the subject-matter. Ball and colleagues have suggested as much in their work (Ball, Hill, & Bass, 2005; Hill, Sleep, Lewis, & Ball, 2007; Ball, Thames, & Phelps, 2008).

As a second example, for an elementary teacher who focuses on computational fluency, CCK would be procedural knowledge of how to conduct the division of fractions. Students could acquire this knowledge through the instruction; they do not need to know why the teacher uses this procedure as long as it is not one of the learning goals for this instruction. Finding an answer for students' why questions about this method is the result of his/her SCK. However, this might be classified as CCK for a high school teacher if s/he has to teach it with its reasoning because procedural reasoning for the division of rational numbers might be one of the learning goals for a specific class meeting. In this respect, a knowledge statement might be called as CCK for one grade level or class, while it might be SCK for another grade level or class.

I have presented literature to define knowledge, to differentiate it from beliefs, and to give examples of teacher knowledge models for mathematics and other subject areas. The final teacher knowledge model presented was Ball and her colleagues' model of *MKT* (Ball, Thames, & Phelps, 2008) as their model frames my study and research questions. After highlighting recent literature and crucial findings, I concluded the knowledge section of this chapter with my working definition of knowledge, differentiated it from beliefs and elaborated on the construct of SCK for my study. The following section delves further into beliefs as a cognitive construct in order to further distinguish beliefs from knowledge.

Teacher Beliefs as a Construct

According to Pajares (1992), teachers' beliefs and belief structures should be the focus of educational research in order for reforms to be achieved in teachers' practices. There have been several empirical studies on teachers' beliefs that had difficulty in conceptualizing and defining beliefs. Pajares (1992) did not explicitly pose a definition for beliefs within his literature review, but referred to other researchers to reveal a convergence of their definitions for beliefs. Regarding these definitions, I can define beliefs from this review as dispositions towards actions and behaviors as a result of previous experiences, and representations of reality with enough valid and true propositions (Pajares, 1992).

Philipp (2007) shared a similar definition within his literature review: "Beliefs [are] psychologically held understandings, premises about the world that are thought to be true" (p.259). He also defined knowledge as "beliefs held with certainty or justified true belief", and added that "what is knowledge for one person may be belief for another" (p. 259).

As I mentioned before, the difference between knowledge and beliefs is not very straightforward. For Pajares (1992), knowledge is dynamic whereas beliefs are more static in terms of their capacity to change over time. While beliefs are inflexible and mostly indisputable truths for believers, as each individual holds beliefs according to the interpretations of their experiences, knowledge is more open to change and dynamic as a consensus of a group of individuals after a possible discussion.

Beliefs are formed early, and sometimes persevere even in light of contradictions emerging from logical reasoning and explanations. Another difference raised by Pajares (1992) between knowledge and beliefs is the level of evaluative and affective nature each hold when a new phenomenon is encountered. Beliefs have an evaluative and affective filtering nature for the new coming information (Abelson, 1979). Even though thought processes such as reasoning and understanding can be in action during the creation of beliefs, beliefs are more effective in terms of filtering thought processes while understanding a phenomenon or interpreting a personal experience, which makes beliefs more evaluative and affective. For example, we might judge the correctness of new information, and claim that it is not correct when it is conflicting with one of our static beliefs even if the new information has been negotiated to be true by a group of people with enough reasonable explanation.

Thompson (1992) oppositely viewed knowledge as more static, and beliefs more dynamic, but she also underlined that knowledge is consensual and beliefs are more convictional. According to her, beliefs could change as a response to life experiences and their interpretations. Secondly, she considered that it is useless to look at knowledge and beliefs as separate cognitive entities within research practices; and added a new cognitive construct as an overarching term to look at knowledge and beliefs together: *conception*. She defined conceptions as “a more general mental structure, encompassing beliefs, meanings, concepts, propositions, rules, mental images, preferences, and the like (p. 130)”. The definition of conceptions included beliefs and cognitive structures that form knowledge structures such as concepts, propositions, rules, and mental images.

Kagan (1992) compared beliefs to knowledge by corresponding the former to opinions and the latter to facts. However, she also admitted that knowledge and beliefs are interrelated by defining knowledge as “belief that has been affirmed as on the basis of objective proof or consensus of opinion” (p. 73). The author defined teacher beliefs in a similar way, as “provocative form of personal knowledge that is generally defined as pre- or inservice teachers’ implicit assumptions about students, learning, classrooms, and the subject matter to be taught” (p. 65). Regarding these two definitions, both knowledge and beliefs are intertwined constructs, and the degree of consensus for its approval determines cognitive entities to be labeled as beliefs or knowledge. In addition, Kagan (1992) presented two generalizations for teacher beliefs: 1) teacher beliefs are associated with a congruent style of teaching across different grade levels and classes, 2) teacher beliefs are mostly stable, and resistant to change. Kagan’s (1992) second generalization demonstrates an agreement with Pajares’ (1992), and contradicts Thompson’s (1992) descriptions of beliefs.

For this study, I follow the definitions given by Pajares (1992) and Kagan (1992); and define beliefs as separate from knowledge and as cognitive entities, which are formed and emerged from individuals' experiences, and interpretations of the happenings around them. I use Pajares' (1992) and Kagan's (1992) definitions as a route for my working definition of beliefs as both authors' definitions made a clear attempt to differentiate beliefs from knowledge. I agree that knowledge is a related cognitive construct to beliefs, but I view and operationalize them separately for my study. Beliefs are more static than dynamic and serve as personal lenses for how people perceive their daily realities (Pajares, 1992). Regarding the evaluative nature of beliefs (Abelson, 1979), teachers' beliefs would influence their daily instructional decisions and actions when they encounter a problem, and when they are pushed to change their practices. To differentiate beliefs from knowledge, I examine the degree to which teachers' statements or observed behaviors indicate opinions rather than facts (Kagan, 1992). Opinions can be identified as they are mostly individualistic, and teachers do not seek or require external validation to support their opinions. Facts on the other hand, can be and often are validated by a group of people who negotiate their correctness and their consensus is an indication that the premise is *known* rather than *believed*.

Teacher Beliefs, Their Change, and Teacher Education Programs

With the emergence of new ideas from research in education, teacher education institutes modified their programs and took actions to accelerate teachers' change in practice. To actualize such changes, changing teachers' beliefs might also be necessary during and after teacher education programs because of the fact that teachers' beliefs interact with how they would interpret teacher education program goals and content and whether they would adapt new ideas in their teaching appropriately (Hollingsworth, 1989; Borko, Mayfield, Marion, Flexer, & Cumbo, 1997; Guskey, 2002).

Fourteen elementary and secondary graduate pre-service reading teachers participated in Hollingsworth's study (1989), which examined the influence of teachers' preprogram beliefs on their knowledge development and beliefs change. The graduate teacher education program in this study focused on Piaget's theory of human development, the constructivist view of learning, and lesson design approaches. Hollingsworth (1989) identified how teachers' beliefs behaved as lenses for their interpretation of graduate program courses as well as their teaching practices later. Moreover, the author also recognized the importance of having teachers who articulated viewpoints that were contrary to the program goals. These discussions forced other participating teachers to defend their new beliefs and prevented them from accepting teacher educators' reform-based behaviors without refection or understanding.

Borko et al. (1997) found similar findings to Hollingsworth (1989), and discussed the importance of challenging teachers' previous beliefs coming from their past schooling or teaching experiences. The authors studied fourteen third grade teachers during a year-long professional development program. Weekly workshops alternated their focus between doing mathematics and reading each week. The workshops aimed to support changes in teachers' assessment techniques and instructional goals. Findings from this study indicated that teachers either ignore, or apply new ideas inappropriately unless their beliefs are challenged. Teachers might not persevere to change their practices even though their beliefs about a new program are questioned. Willingness to change could be another factor for a teacher to experiment with different strategies in his/her classroom.

Regarding the importance of beliefs for the adaptation of new philosophies, perspectives embedded within any reform movements, researchers also investigated teacher education programs to determine effective experiences in changing teachers' beliefs before starting to teach (Hollingsworth, 1989; Grant, Hiebert, & Wearne, 1998; Peressini, Borko, Romagnano, Knuth, & Willis; 2004). These studies attempted to understand how teachers' prior beliefs affect their teaching practices after completing their programs.

Cooney, Shealy, and Arvold (1998) studied belief structures of four pre-service secondary mathematics teachers, how these structures change while they are going through their teacher education program, and to what extent these changes result from experiences within the teacher education program. In their study, the researchers observed and interviewed PSTs during the last year of the program, which included a curriculum course, a methods course, student teaching and a seminar. During five interviews, the researchers questioned participants' belief structures, their perceptions of the professors' view of mathematics, their experiences within the program, and their interpretations. The study indicated different profiles of experiences and belief changes for each teacher with respect to their personal traits. For example, one of the participants was more open to others' opinions and trying out new things, which enabled a belief change for him about teaching mathematics. On the other hand, another participant was not as open to different perspectives, which challenged his beliefs about teaching mathematics from the beginning. Having an authoritative view for teaching mathematics did not help him to adapt new teaching approaches into his practice, but assimilate them.

Vacc and Bright (1999) specified the extent of the teacher education program, and examined whether a teacher education program focused on PSTs' adaptation of Cognitively Guided Instruction is effective in changing teachers' beliefs about mathematics instruction. For the study, pre-service elementary mathematics teachers attended a mathematics methods course, weekly seminars, and participated in an internship and student teaching. The authors presented survey findings of all 34 PSTs, and provided details for two of them. Even if the authors found that teachers changed their beliefs and viewed CGI and constructivist views as appropriate for their classes, their practices did not always reflect these changes. We cannot also be certain about the quality of changes in teachers' beliefs and practices. While these changes could be permanent and fundamental for some teachers, they might be superficial for others. In a way, Vacc and Bright's study (1999) revealed changing PSTs' beliefs is difficult because of a lack of teaching experience.

Rather than studying teachers' beliefs and teaching practices in general, Peressini, Borko, Romagnano, Knuth, and Willis (2004) narrowed down their research topic, and examined the experiences of two pre-service mathematics teachers during their teacher education, how their experiences influenced their views about mathematical proof, and which way these views were enacted in their classrooms after they completed their programs. The authors chose to study this phenomenon within a situative perspective, by which multiple social contexts such as classroom social practices, discourse patterns, participation by students and the teacher, personal identities are taken into account. A case study design was used as the methodology; and two teachers' experiences during college, student teaching, and first year of teaching were described in depth. The narratives of teachers' experiences showed the emergence of a contradiction regarding the definition of mathematical proof in different settings. In their college mathematics course, two participating teachers defined and practiced mathematical proof with certain and rigorous techniques to justify the correctness of a mathematical statement. While teaching however, they did not approach it the same way. Rather, they used proof as a means to help students understand mathematical content. This finding indicated how novice teachers develop their beliefs; adapting some teaching practices while ignoring others, even if the adapted practices are in conflict with the intended purposes of their teacher education programs.

Frameworks for Beliefs about Mathematics

There have been several frameworks used to identify mathematics teachers' views and conceptions of mathematics (Skemp, 1978; Ernest, 1989; Lerman, 1983; cited in Thomson, 1992). For example, in his framework, Skemp (1978) presented two types of mathematics: instrumental and relational mathematics. Teachers focus on the use of certain procedures and strategies in order to solve mathematical problems when they view mathematics instrumentally. On the other hand, with a view of relational mathematics, teachers emphasize conceptual learning and conceptual connections between different strands of mathematics. Instrumental mathematics is easier to understand and to assess; achievement goals are easier to attain; and producing an answer is quicker. Relational mathematics is more adaptable to new tasks and problems, easier to remember, and increases intrinsic motivation as it generates an internal force in reaching the achievement goals. Viewing mathematics relationally might be a challenge for many teachers if it has not been learned and experienced in this way while they were learners. Skemp (1978) argued that, to be able to construct mathematics in a relational perspective rather than the instrumental perspective, teachers have to be given opportunity and time to explore mathematics conceptually prior to their teaching experience.

Ernest's (1989) three views about the nature of mathematics can be related to Skemp's (1978) view of instrumental and relational mathematics: 1) Problem solving view of mathematics, 2) Platonist view of mathematics, 3) Instrumental view of mathematics. In the problem solving view of mathematics, the nature of mathematics is conceived as a human creation that can be constructed through exploration of mathematical problems. According to the Platonist view, mathematics is perceived as a static entity that has been discovered. The mathematics learner is expected to acquire this static knowledge, consisting of mathematical concepts and its structure with relationships among concepts. Skemps' (1978) definition of relational mathematics can be seen as an amalgamation of Ernest's (1989) problem solving and Platonist views of mathematics. Ernest's instrumental view of mathematics emphasizes a traditional view, by which mathematics is viewed as a tool bag full of procedures, facts, algorithms and rules that have to be mastered. This conception of mathematics largely resembles Skemp's (1978) description of instrumental mathematics.

Lerman (1983; cited in Thomson, 1992) proposed yet another similar categorization of types of mathematics: 1) absolutist where mathematics is abstract, certain, value free; and 2) fallibilist where mathematics is uncertain, and it has to be created. Compared to Ernest's (1989) classification for the views about the nature of mathematics, Lerman (1983; cited in Thomson, 1992) categorized mathematics according to its user's epistemological view by putting stress on whether mathematics is certain or not, and how it might be developed. While having either the absolutist or fallibilist view of mathematics does not influence teachers' practice and students' gains from instruction, Thompson (1992) discussed that teachers' views about the nature of mathematics according to Ernest's (1989) categorization does seem to be a factor on teachers' instructional decisions.

Out of these three frameworks for beliefs about the nature of mathematics, I employed Ernest's (1989) framework for my study as I see it as more extensive than others (Lerman, 1983; cited in Thomson, 1992; Skemp, 1978) in terms of the distinction between each category. Even though both Lerman's and Skemp's classifications for beliefs about the nature of mathematics are similar to the categorization given by Ernest (1989), their definitions might still form some sub-categories. For example, I see aspects of both the problem-solving and Platonist views of mathematics (Ernest, 1989) within the relational view of mathematics by Skemp (1978). In other words, categories given by Ernest provide more fine-grained options to classify teachers' beliefs about the nature of mathematics separately. Second, the definitions of each belief about mathematics under Ernest's (1989) framework are more linked with the context of teaching. Lerman's framework (1983; cited in Thomson, 1992), on the other hand, is more about the epistemology of mathematics; and does not demonstrate clear connections to the context of teaching.

Frameworks for Beliefs about Teaching Mathematics

Kuhs and Ball (1986; cited in Thompson, 1992) created a categorization of orientations for teaching and learning mathematics. They presented four viewpoints for teaching mathematics: 1) learner-focused, 2) content-focused with an emphasis on conceptual understanding, 3) content-focused with an emphasis on performance, and 4) classroom-focused. Thompson (1992) defined these views about teaching as it follows:

In the learner-focused view, teachers focus on students' personal sense making of mathematics they experience while interacting with mathematical tasks and activities. Students would be guided to construct their own mathematical knowledge with the support of a teacher in the classroom. In content-focused view with an emphasis on conceptual understanding, teachers prioritize mathematical content, concepts and relationships among them. At the same time, s/he emphasizes comprehension of main mathematical concepts with their connections to the big picture. On the other hand, students' acquisition of rules, procedural skills, problem-solving techniques with efficiency, the use of certain mathematical terms, and their automation are aimed by the teacher in content-focused view with an emphasis on performance. Finally, teachers with a classroom-focused view of teaching mathematics tend to focus on practices of effective teacher behaviors defined and described within the process-product studies, such as managing classroom effectively, pursuing higher expectations from students, transmitting clear structure for the content, use appropriate assessment techniques and feedback.

In their study, Grant, Hiebert and Wearne (1998) used a simplistic framework to categorize teachers' beliefs about mathematics and teaching mathematics. Participant teachers were classified either as having an emphasis on students learning skills and algorithms or on learning concepts and processes. The authors also labeled teachers' beliefs about teaching mathematics according to whom the responsibility for instructional decisions concerning what and how to teach/learn was allocated: to the teacher, or to students. Compared to Ernest (1989) and Kuhs and Ball (1986; cited in Thompson, 1992), Grant and colleagues' framework seems to be more limited in its usefulness in classifying teachers' views about mathematics and teaching mathematics. Regarding these limitations, I employed Kuhs and Ball's framework (1986; cited in Thompson, 1992) in my study to categorize participant teachers' beliefs about teaching mathematics. Having four options also allows me to differentiate teachers more with respect to their teaching related beliefs.

Related Research on Beliefs

Thompson (1984) examined the relationship between teachers' views about mathematics, mathematics teaching and their instructional practice through a case study approach. Three junior high school mathematics teachers were the participants of the study. For two weeks, the participants' classes were observed, and for the next two weeks, participants were interviewed after Thompson's daily observations. After a discussion of the relationship between each teacher's views about mathematics, their beliefs and instructional behaviors, a cross-sectional analysis was used to identify the properties of the system describing the differences among teachers' views, their integrated-ness and reflectiveness. Thompson (1984) concluded that the relationship between teachers' views about mathematics and their instructional behaviors is complex because many decisions and beliefs that are not related to mathematics have an interactive effect on this relationship. The author also demonstrated the relationship between instructional behaviors and teachers' views about mathematics by stating "teachers develop patterns of instructional behaviors that may be manifested from their consciously held beliefs and preferences" (p.173).

In researchers' examination of how mathematics teachers change their beliefs on mathematics and teaching mathematics, they discovered some structural changes in beliefs as well as knowledge development, as the authors approached beliefs and knowledge as intertwined and interconnected constructs (Grant, Hiebert, & Wearne, 1998; Cooney et al., 1998; Vacc & Bright; 1999). For example, Grant and colleagues (1998) found that changes in teachers' beliefs help them to develop mathematical and pedagogical knowledge. Vacc and Bright (1999) also speculated that teachers' theoretical knowledge, combined with a lack of teaching experience, creates an inconsistency between what they believe and how they teach in classroom. With respect to these two studies, teacher knowledge is considered and posited to be a reason and result for teachers' change in beliefs. Even though there are studies discussing the effect of teachers' beliefs about mathematics and teaching mathematics on the development of teacher knowledge, these studies do not indicate a purposeful attempt to make the distinction between knowledge and beliefs clear. This study, on the other hand, attempts to make this distinction clearer, and focuses on the examination of the influence of teachers' beliefs about mathematics and teaching mathematics on a specific type of teacher knowledge: Specialized Content Knowledge (Ball, Thames, & Phelps, 2008).

For this study, teachers' beliefs about mathematics are identified and categorized according to Ernest's (1989) three views about mathematics; and their beliefs about teaching mathematics are classified with respect to Kuhs and Ball's (1986; cited in Thompson, 1992) four views about teaching mathematics. While forming these categorizations, it is possible each teacher might have multiple beliefs about mathematics and teaching mathematics. In addition, beliefs may reveal a natural link between teachers' beliefs about mathematics and their beliefs about teaching mathematics (van der Sandt, 2007). For example, van der Sandt (2007) considered that Kuhs and Ball's *learner-focused teaching belief* would follow Ernest's *problem solving belief about mathematics*; *content-focused teaching belief with an emphasis on conceptual understanding* would follow *Platonic belief about mathematics*; and *content-focused teaching belief with an emphasis on performance* would follow *instrumental belief about mathematics*. In this section, I have framed the theory for beliefs about mathematics, and teaching mathematics for my analysis, but the methodological and analytical decisions for data coding and labeling are discussed in greater detail in Chapter 3.

This study aims to examine pre-service mathematics teachers' SCK, its development, and the influence of their beliefs about mathematics and teaching mathematics while they are using a specific technology, dynamic geometry software. As such, the final section of my review of the literature focuses on my conceptualization of technology, its definition for this study, and the review of the recent literature on teacher's knowledge development in technology-integrated environments.

Instructional Technology and Knowledge

With the improvement in information technologies, people in the field of education look for different ways and purposes to integrate emerging technologies into the instruction in order to facilitate students' learning gains. For example, Handal and Herrington (2003) described three different ways to use electronic technologies and computers: 1) technology as a tool, 2) technology as a tutor and 3) technology as a tutee:

Technology can be used as a tool in class to facilitate tasks peripheral to students learning, such as note taking or data saving. Computer software programs such as MS Excel or MS Word are viewed as *tools*. For many years, calculators have been also only used as a tool for computation and justification of findings from basic operations. Computers can also be used as a *tutor* for students' learning, for example, of problem-solving skills. In the tutor view, the computer presents a specific problem, gives feedbacks to the students' responses, and guides them with respect to the level of their knowledge and skills. Finally, computers can also be viewed as a *tutee*. In this category of technology use, students program computers and write computational commands in order to solve a mathematical task. LOGO was one of the first examples in which computers were used as a tutee. With this program, learners guide an imaginary turtle on the screen by programming commands for a specific task.

Considering these three different uses of technology by Handal and Herrington (2003), technology can be defined as an instrument that supports users' behavioral tasks peripherally, or that changes their environment and learning substantively. When technologies are used in a specific manner relating to the second part of my definition, they can be classified as cognitive tools (Jonassen, 2003).

Cognitive Tools

Cognitive tools are mental and computational devices that enhance and extend humans' thinking processes and cognitive capabilities, support knowledge construction, and release the cognitive burden through its expertise and possibility of intellectual partnership with it (Jonassen, 1992). Jonassen (1992) described cognitive tools as technologies that support learning through construction of knowledge and generative processing. The author defined generative processing as the cognitive activity learners use to relate the incoming information to their previous knowledge with the use of cognitive tools. With respect to this definition, each individual constructs his/her knowledge with cognitive tools differently. In cognitive tools, knowledge is not approached as an external reality that can be accessed by anyone in the same way.

Cognitive tools also create an intellectual partnership with users so that both the expertise of the tool and the user are shared and facilitated (Jonassen, Carr, & Yueh, 1998). The joint system of learning provides a cognitive advantage by off-loading unnecessary memorization tasks from user to computer. This cognitive release allows the user to be occupied with higher order thinking and deep cognitive processing skills (Kim & Reeves, 2007). In other words, cognitive tools reduce the extraneous cognitive load, and create a space for germane cognitive load enabling meaningful learning (Sweller, 2007).

Specific advantages of cognitive tools for learners are: 1) engagement with higher order skills; 2) durable encoding and retrieval of information; and 3) mind extensions. According to Kim and Reeves (2007), cognitive tools should push users to be occupied with higher cognitive skills which require deep processing, rather than lower cognitive skills such as memorization. It might be advantageous as long as the task embedded within the cognitive tool requires learners to deal with higher-order cognitive and thinking skills. Cognitive tools also contribute to the encoding and retrieval of information for a long period of time and with more capacity (Mayes, 1992). Meaningful learning opportunities given by the use of cognitive tools might be the reasons for this advantage. In addition, according to Jonassen, Carr and Yueh (1998), cognitive tools work as mind extensions by doing unnecessary memory tasks for the user. The user does not need to utilize his/her cognitive capacities for computational or representational tasks, but understanding and connections of these tasks.

Although researchers defined cognitive tools as innovations that bring expertise to the task and make the distribution of cognition possible, thinking, processing, understanding and interpreting are still the job of individuals, not of the computer. Cognitive tools are still unintelligent tools, which can create an environment for the user to use his/her intelligence in a better way (Jonassen, 1995). However, if a user approaches the tool from a traditional standpoint, and waits for the transmission of the knowledge from the computer rather than its construction, then the cognitive tool does not make a difference in the user's learning.

The use of cognitive tools in a classroom requires a change in instructional approaches and role adaptations from teachers and students. In an environment where students use computers as cognitive tools, the teacher needs to develop and instill different classroom and social norms. Formation and application of these norms may be a challenge for many teachers. Further, the nature of classroom assessment has to change as well. If the teacher employs cognitive tools during the instruction, and expects students to learn with these tools, then the students should be required to use the same tools for the assessment of their learning. Otherwise, there might be a difference between what is provided and what is expected (Kim & Reeves, 2007).

Finally, cognitive tools are still design dependent. Technological affordances of cognitive tools as well as the quality of the accompanying task would make it advantageous in some settings, and disadvantageous in others. For example, dynamic geometry software might be very beneficial for students to understand construction of geometrical figures and polygons as long as the tasks and guidelines given with the technology make sense. On the other hand, the same technology might be confusing for students' understanding of irrational numbers because this kind of technology is limited to represent a line segment where its length is equal to pi. Pi is defined as the ratio of a circle's circumference to its diameter. We also know that either the circumference or the diameter of the circle has to be an irrational number, otherwise pi cannot be an irrational number. The software can measure the circumference and the length of a circle's diameter with decimals. While the software presents an approximation for circumference and the length of the diameter, students' might consider it as precise measurements of two rational numbers. Regarding that, constructions depending on irrational numbers might be confusing for students.

Considering a definition of cognitive tools that emphasizes knowledge construction (Jonassen, 1992), cognitive tools can best be supported by the use of constructivist instructional approaches. This type of tools cannot provide advantageous learning results when paired with traditional approaches such as drill, memorization and practice. Second, the quality of the task is as important as the affordances of the technology: teachers should consider and prepare authentic mathematical tasks that demand higher-order thinking and deep processing skills that leverage the distribution of cognition between the user and the tool. As a result of this distribution, students do not spend their times with lower cognitive skills such as computation, but focus on higher cognitive skills such as interpretation, analysis, evaluation and synthesis. Students can have such learning opportunities emphasizing higher cognitive skills because technology makes and operates unnecessary, and sometimes time consuming, tasks such as computation or graphical representations for them.

Cognitive tools can also support teachers' content knowledge development if used to explore and solve challenging mathematical problems. During this process, they might need to externalize their conceptual structure about the problem, identify the major concepts and procedures, and the meaningful links among them. One way to achieve content knowledge development with cognitive tools is "Learning by Design". To learn a technology by design, teachers might be given an opportunity to plan lessons that utilize cognitive tools during instruction rather than using prepared lessons or applets (Kim & Reeves, 2007). For example, teachers might create an applet with GSP so as to create a challenging mathematical task. The design procedure forces teachers to externalize their available content knowledge, and to use that knowledge to design an applet.

Experience with cognitive tools during the design process also extends teachers' content knowledge. The technological capabilities of these tools could potentially amplify their previous knowledge, or provide new content knowledge from this experience. The process of designing and constructing materials would enable designers to understand the subject they are teaching more deeply than those whose thinking was constrained by the tools (Jonassen, 1995).

Current mathematics education technologies include computers, computer software specific to mathematics (for example GSP or Fathom), graphing calculators, Smart Boards, and several other electronic utilities (e.g. mathematics applications for smart phones). In the middle and high school, it is expected students are introduced to and become facile in using graphing calculators and a variety of computer software, both of which have potential to amplify students' mathematical conceptions. These technologies are not new for mathematics teachers, but the purpose of using these technologies is problematic. Because of the accountability movement since 2000, mathematics teachers inclined to use technologies for testing rather than students' learning. Dynamic Geometry Software (e.g. GSP or GeoGebra), MS Excel, Semantic Networks, modeling tools and Micro-worlds are examples of software that could be implemented in classrooms as cognitive tools. In the next section, I focus on the description of dynamic geometry software because of its role in my study.

Dynamic geometry software.

Dynamic Geometry Software (DGS) is an effective innovation used with computers that have the potential to enhance learners' conceptual understanding of geometrical relationships, conjecturing and argumentation skills. With DGS, students can "create drawings, make measurements, and drag the drawing while the drawing maintains the dependent relationships that were formed in the construction of the drawing" (Smith, 2010, p. 4).

DGS is problem-solving tools that convert a static representation into a dynamic one. Its design features can encourage students to activate their cognitive processes (Santos-Trigo & Cristobal-Escalante, 2008). It provides non-traditional ways to understand mathematical concepts by allowing users to see the relevancy of a statement through many visual examples in a few seconds (Marrades & Gutierraz, 2000). In addition, this type of software enables users to measure particular elements (e.g. sides or angles) of a geometrical object and to identify patterns and constants related to the object. A hidden property of a geometrical figure can be explored and discovered through conjectures, and eventually explained or proven formally. In this sense, users firstly reason inductively, formulate a pattern, consider it as a conjecture, and test the conjecture through trials. Then they reason deductively in order to generalize their observations in the dynamic geometry environment. Such a view of geometry conflicts with its rigid view that emphasizes deductive proof. However, it improves students' explanation skills, and thus, their understanding of geometrical concepts by reducing the cognitive load coming from basic computations and operations (Brown, 2010; Robinson & Burns, 2009; Liu & Bera, 2005).

After DGS became prevalent in K-12 mathematics classes and was found to be effective in enhancing students' conceptual understanding of geometrical relationships, conjecturing and argumentation skills, teacher education programs began to include these technologies in the training of pre-service and in-service teachers. In other words, both practicing and prospective teachers are expected to know some specific mathematical technologies and methods to use them effectively in classroom. Therefore, the concept of teacher knowledge has required modification in order to account for the emergence of new instructional technologies and related 21st century skills expected from teachers. The following section presents and discusses the reconceptualization of teacher knowledge in light of the necessity of technology use for the mathematics instruction.

Reconceptualization of Teacher Knowledge for 21st Century Needs

Teacher education programs initially focused on teachers' technology knowledge development through a techno-centric approach by which learning affordances and constraints of technologies becomes the basis for teacher preparation and further professional development. The techno-centric approach separates technology, content and pedagogy courses and does not enable teachers to integrate technology in a proper way.

Harris, Mishra and Koehler (2009) identified the lack of content and pedagogy in this approach as its main weakness. In this approach, teachers might learn to use technology, but not learn how to integrate technology for specific mathematical and pedagogical goals (Harris et al., 2009). Koehler and Mishra's framework (2005) for Technological Pedagogical Content Knowledge (TPACK) has been a reaction to the techno-centric approach, and provided an integrated knowledge structure that included technology.

The TPACK framework has become widely accepted as an appropriate model and approach for professional development that would organize activities and form opportunities for teacher learning within the consideration of the interconnection between specific content, pedagogy with technology. Its use was strengthened due to its focus on learners' deep conceptual understanding instead of drill and practice methods (Bowers & Stephens, 2011).

Technological pedagogical content knowledge.

With the emergence of new technologies, teachers' attention has shifted to students' thinking, curriculum and pedagogical decisions for the emergence of technology in classroom (Niess, 2011). A new framework for conceptualization of teacher knowledge was developed to support teacher preparation programs in engaging teachers with appropriate technology integration. Researchers cited the importance of integrating technology knowledge with pedagogical knowledge and content knowledge, similar to the way in which Shulman (1986) proposed in his development of PCK (Koehler & Mishra, 2005; Niess, 2011) from the integration of pedagogical knowledge with content knowledge. This exploration resulted in the development of the Technological Pedagogical Content Knowledge framework. The first acronym used for the framework was TPCK. During a two-day conference for the 9th Annual National

Technology Leadership Summit, participants discussed suggestions for a new name for TPCK. After scholarly discussions, the acronym of TPCK was modified into TPACK (Thompson & Mishra, 2007).

The representation of the construct as well as the framework went through developmental processes: the first graphical representation included technology, content, learning and teaching as three sets intersecting, and their intersection indicated the TPACK. Koehler and Mishra (2005) extended these three sets into seven components in the handbook of Technological Pedagogical Content Knowledge (*see Figure 2.2*): technology knowledge (TK), pedagogy knowledge (PK), content knowledge (CK), technological content knowledge (TCK), pedagogical content knowledge (PCK), technological pedagogical knowledge (TPK), and technological pedagogical content knowledge (TPACK).

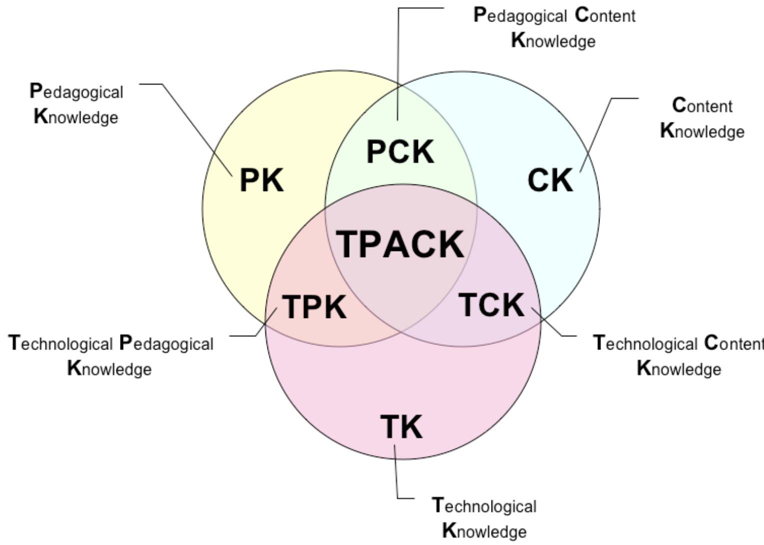

Figure 2.2: Representation of the TPACK Framework (Koehler & Mishra, 2005)

Harris and his colleagues' (2009) articulated and clarified the constructs of the TPACK framework. According to their understanding, content knowledge consists of knowledge of concepts, theories, organizational frameworks, methods of evidence and proof. Pedagogical knowledge encompasses knowledge of educational purposes, goals, values, strategies, student learning and needs, classroom management, instructional planning, implementation and assessment. Their definition of technological knowledge is fluid and not as clear as content and pedagogical knowledge. This is largely needed as the definition needs to be able to change and adapt in light of the frequent development of new technologies. However, they did not restrict the notion of technology to Information Communication Technologies (ICT) or electronic technologies. For example, they also considered whiteboards as a technology used by teachers. Koehler and Mishra's (2005) definition of technology also encompasses commonplace technologies as overhead projectors, blackboards and books.

Because one of my research questions examines Technological Content Knowledge (TCK), I describe TCK more in detail here. TCK "is useful to describe teachers' knowledge of how a subject-matter is transformed with the application of technology" (Koehler and Mishra, 2005, p. 134). The definition of TCK assumes that both technology and content mutually influence and constrain each other. To show this two-directional interaction, Harris et al. (2009) gave examples from the developments in technologies and how these developments affected and enabled new discoveries in different disciplines such as medicine and chemistry.

Doerr and Zangor (2000) described two types of knowledge development for technology knowledge and technological content knowledge. Any user, teacher or student, would first interact with the tool, and this interaction would allow the user to make sense of the tool in terms of its capacity and limitations. The authors described this act as developing *meaning for the tool*. In a way, meaning for the tool supports the user's development of technological knowledge. The interaction with the technological tool also facilitates understanding of concepts as well. During this process, the user gains *meaning with the tool*. This experience contributes to the development of users' technological content knowledge.

The operational definition of TCK in Richardson's (2009) study looks more closely at the concept of *meaning with the tool* (Doerr & Zangor, 2000). In this study, 20 in-service eighth grade mathematics teachers went through a professional development program for one year which guided teachers in learning algebra using technology. The author presented findings for three different activities and showed some evidence that teachers developed their TCK more than their TPACK and PCK. This study did not focus on teachers' learning of technological affordances for a specific program. My study tried to overcome this limitation with its focus on geometry and the GSP.

Harris et al. (2009) defined the overarching concept of TPACK as the result of multiple interactions among pedagogy, content, technology and contextual knowledge. Even though Harris et al. (2009) defined and perceived TPACK as a construct resulting from interaction among knowledge components, Groth, Spickler, Bergner and Bardzell (2009) considered TPACK as an evolving construct that does not include a final definition to apply or to use. Expected skills for this knowledge development are comprehension and communication of representations of concepts, using technologies and technology specific pedagogical decisions for different learners' needs.

Developing TPACK through teacher education.

TPACK has been considered as an appropriate framework to guide researchers and teacher educators for the preparation and evaluation of pre-service and in-service programs. Both Lee and Hollebrands' (2008) and Niess' (2005) studies show this purpose and orientation. In Lee and Hollebrands' (2008) study, teachers practiced solving mathematical tasks with technology as learners, and later reflected on the capabilities and constraints of the technology they experienced in considering the use of technology for teaching mathematics in the future. Niess (2005) also presented research from a content-based technology integration course. Within this course, teachers progressed through a program emphasizing research and mathematical problem-based technology integration activities and PCK development simultaneously. Both Niess' (2005) and Lee and

Hollebrands' (2008) studies enabled PSTs to restructure their TPACK by their learning experience with technologies. However, Niess also added that having limited teaching experience did not support PSTs to understand how to mesh technology, pedagogy and content knowledge appropriately. One of the teachers in Niess' study (2005) focused on technology integration from the beginning of her student-teaching experience, but could not make a clear connection between mathematics and technology while teaching. The author linked this lack of connection to the teacher's limited teaching experience and knowledge about teaching and classroom management.

Many research studies have been conducted to investigate how teacher education programs prepare teachers with the necessary skills to teach with technology. Researchers investigated the interactions among knowledge components and which experiences supported PSTs' development of TPACK (Groth et al., 2009; Koehler & Mishra; 2005; Niess, 2005). For example, Koehler and Mishra (2005) argued TPACK development requires the design of a curricular system that appreciates multidimensional relationships among the technology, content and pedagogy components. Regarding this argument, the authors claimed that teachers develop their TPACK if a learning technology is employed by a design approach in which ill-structured authentic problems serve as the context for teacher learning. In this study, participants were responsible to design an online course. Pedagogical decisions in a teacher education program were hypothesized as a factor for the TPACK development of pre-service mathematics teachers. The ill-structured problems, which reflect the complexity of the real world, provided a change for participating teachers from a discomfort to accomplishment and recognition of technology for learning. Quantitative analyses revealed the design approach resulted in practical changes in each combination of technology, content and pedagogy components.

Whereas Koehler and Mishra (2005) focused more on the interaction between the pedagogy and technology knowledge, Niess' study (2005) demonstrated the importance of the interaction between technology and content knowledge for pre-service teachers' TPACK development. The graduate program described in this study concentrated on research-based teaching and learning, campus-wide instructional practices, subject specific content, subject specific feedback from faculty, and subject specific technology courses. Niess (2005) concluded that PSTs' experiences with technology during the development of their subject specific content knowledge within a one-year program enable their development of TPACK. The study provided more findings for science teachers than mathematics teachers. Moreover, the results of this study were more related to technological pedagogical knowledge rather than TCK.

Ozgun-Koca, Meagher and Edwards (2009) studied the development of pre-service secondary mathematics teachers' TPACK by looking at their lesson plans for technology-integrated instruction. Throughout their lesson plans, teachers used technology in a superficial way rather than incorporating it in a consideration of the capabilities of technologies and opportunities that might occur from an appropriate usage. The PSTs' lack of technological knowledge inhibited their ability to make pedagogical decisions that would overcome the limitations of the technology. The study also showed that teachers did not see technology as a means to use inquiry-based instruction. Rather, they considered that technology was providing excessive amount of information for students rather than letting them construct. PSTs in this study focused on the development of their content and then pedagogical content knowledge, but somehow did not integrate technology onto these two components. Because of that, teachers could not develop TCK and TPACK as much as CK and PCK.

Another experience required for PSTs' development of TPACK is learning experience with technology for the content they teach (Bowers & Stephens, 2011; Niess, 2005). Without such experience, PSTs are inclined to use technology superficially and peripherally to the curriculum content, and they could not perceive technology as mediums which change and enhance students' learning and understanding of mathematics. Such experiences also provide an opportunity for teachers to understand both capabilities and limitations of a specific technology for specific mathematical content (Bowers & Doerr, 2001).

TPACK was approached both as a framework to orient teacher education programs and professional development projects, and as a knowledge framework, which specifies expectations from teachers for the 21st century needs. While the orientation for teacher preparation and development might be effective and clear, the framework for teacher knowledge was not very elaborative the same as other knowledge frameworks (e.g. the MKT framework for mathematics teacher knowledge). Furthermore, within the TPACK framework, technology was considered as a new knowledge component rather than a factor influencing teacher knowledge. Regarding my first research question about the influence of technology on teachers' SCK development, I also reviewed literature that focused on the examination of technology as a factor affecting teacher knowledge.

Technology and MKT Development

As my work specifically examined the development of SCK through the use of technology, I next searched the literature for studies that investigated this phenomenon. This literature is more limited, which indicates my work would address a gap in the current literature.

Silverman and Clay (2009) studied the effect of an online collaborative environment on pre-service and in-service K-12 teachers' MKT within an online Geometry and Algebra content course. In the online collaboration environment, teachers privately posted their thinking processes concerning the solution of the problem. Next, these private postings were injected into the group discussion by the authors who served as the moderators for the online environment where teachers interacted with each other in the process of making sense of solutions as well as in discussing pedagogical decisions. The study revealed the online discussions allowed teachers to share their thoughts on teaching practices and pedagogical insights. The different postings also provided teachers with different perspectives and strategies for the same mathematical task. These findings might indicate the use of technology as a social opinion-sharing environment can enable teachers to share their mathematical ideas and strategies more comfortably, to recognize the articulation of their thought processes during the solution of a mathematics problem, to be aware of different strategies and ideas for the same problem, and to receive feedback on their strategies and ideas. In other words, an online collaborative environment help teachers develop their PCK, more specifically KCS, by looking at the ideas and strategies by others in the classroom.

Silverman (2012) reexamined how the participation of teachers in online discussion environments and online problem solving correlate with their MKT development. An online graduate mathematics education course on algebra and algebraic reasoning was the research site for the study. Unfortunately, the authors did not specify whether the participating teachers were practicing teachers going through the online course professional development or PSTs looking for a graduate degree. Similar to his previous study, in the online asynchronous collaboration environment, PSTs were asked to post solutions for the assigned open-ended mathematical task, or post their ideas about the solution of the task before finalizing their solutions. After this individual sharing phase, participating teachers were expected to interact in pairs or as small groups to discuss similarities and differences among the presented solutions. Later, the instructor of the course orchestrated an online whole-class discussion around teachers' solutions and the instructional objectives. The study showed social networking among participants helped their MKT development. However, the authors also found that both participants who were initially categorized as experts in mathematics and others who interacted with these experts in mathematics developed their MKT. Teachers who received more support through their interactions improved their MKT more than others. Even though this study presented important findings regarding the characteristics of online collaboration that influence the development of teachers' MKT, it does not address how or which type of technology can trigger or limit this development.

Overall, SCK has been studied in the literature to differentiate it from CCK (Hill, Schilling, & Ball, 2004), to identify possible interaction with other domains of knowledge (Bair & Rich, 2011), and to explore its development through teacher education programs (Morris et al., 2009). Even less has been done in examining SCK development through the use of technology. Silverman and his colleagues studied the development of MKT (Silverman & Clay, 2009; Silverman, 2012), although it did not focus on SCK. Moreover, this work did little to further our knowledge as to the type of technology beneficial in promoting MKT development.

Frameworks for Beliefs about Technology

As mentioned before, teacher beliefs about their content and/or teaching were found to be influential on their decisions to take action in class as well as their perspectives in order to adopt an educational reform in their classroom (Hollingsworth, 1989; Borko, Mayfield, Marion, Flexer, & Cumbo, 1997; Guskey, 2002). Teachers' beliefs about technology are also factors affecting their pedagogical decisions to adopt educational reforms expecting teachers to use technology effectively (Cavin, 2008). Teachers' beliefs about technology and beliefs of teaching with technology are influenced by the timing of their experiences (i.e. before, during or after pre-service teacher education programs) in learning content through the use of technology (Dawson, 2007; Ertmer, 2005; Groth et al., 2009; Ozgun-Koca et al., 2009; 2011).

Groth et al. (2009) discussed two types of teacher beliefs about technology in mathematics classes. In the first view of technology, the use of technological tools, such as the capabilities of calculators, are used to ease the cumbersome hand calculations. This belief about technology does not change students' conceptual understanding and their learning. In the second type of teacher belief about technology, technological tools changes what is taught and what is learned with the integration of technology.

Groth and colleagues' study (2009) presented the implementation of a lesson plan by a teacher to solve systems of equations by matrices. The instructions given by the teacher guided students to find the inverse of the first matrix, to multiply the inverse matrix with the matrix at the right hand side of the equation in order to find the solution for the problem. To find the inverse of the matrix and to multiply the two matrices, students were told to use the graphing calculators. This instructional order forced students to use technology only for computations. The knowledge of multiple representations and conceptual connections among these representations might be missing for this teacher. This lack of knowledge might have influenced the teacher's perception of how to use the graphing calculator effectively. The lack of experience with the graphing calculator as a mathematics learner might have been another reason for the teacher's beliefs about technology. Learning experience with technology during their teacher education might be supportive for a teacher to view technology as resources that change what is taught. This belief was the second type of teacher belief about technology in the framework by Groth et al. (2009).

Chen (2011) also proposed two types of teacher beliefs about technology: 1) instrumental and 2) substantive beliefs. Teachers who hold instrumental beliefs approach technological devices as tools to improve the efficiency of their instruction without considering whether there is any modified influence on students' cognitive processes or learning. On the other hand, teachers who hold substantive beliefs perceive technology as an aid for students' learning and understanding. Substantive beliefs include the belief that technology can create a new medium in which learners and technological devices engage in reciprocal interactions, resulting in stronger student understanding of mathematical concepts. These two types of beliefs about technology by Chen (2011) show similarity to two types of beliefs about technology by Groth et al. (2009). While a teacher would only view technology as a tool for faster and efficient computations with the instrumental belief, this teacher would start to consider technology as resources to change what is taught with the substantive belief about technology.

Ozgun-Koca, Meagher and Edwards (2009) studied how PSTs' TPACK emerged during their methods classes and field experience which intended for teachers to use advanced digital technologies in the design and practice of technology-enhanced instructional materials. Particularly, TI Inspire was extensively used and focused for the methods course. The authors found that PSTs need to change their perspective from learners of mathematics to teachers of mathematics in order to understand how to integrate technology more effectively to the mathematics instruction. In addition, learning experiences with technology primarily helped secondary PSTs view the calculator as a mediating tool for conceptual learning and knowledge development. However, their student-teaching experiences reversed their perspectives from calculators as a mediating tool for learning to a tool that inhibited students' learning of basic concepts. In other words, teachers' beliefs about technology changed from substantive to instrumental (Chen, 2011) with the addition of teaching experience.

I chose Chen's (2011) framework for technology-related beliefs because the author manages to capture two common beliefs about technology by teachers with this framework. Further, I examined how approaching technology as a cognitive tool compared to approaching technology as a tool only for computation might influence PSTs' knowledge development. Chen's framework and two types of beliefs about technology allowed for this examination. A correspondence can be made between substantive type of beliefs and the use of technology as cognitive tools; and a second correspondence would appear between the instrumental type of belief and the traditional use of technology only for computational purposes.

Researchers have documented a relationship between teachers' beliefs about technology and their beliefs about mathematics (Tharp, Fitzsimmons, & Ayers, 1997; Schmidt, 1998). For example, Tharp, Fitzsimmons, and Ayers (1997) found that teachers who view mathematics as a rule-based discipline do not favor calculator use in their classes. They also discovered teachers who perceive mathematics as a rule-based discipline focus on students' emotions, while the teachers who do not view mathematics in that manner focus on students' conceptual understanding.

A related investigation of mathematics teachers' beliefs is the consideration of when the introduction of technology is appropriate. While some teachers believe that content should be taught prior to an application with a technological tools, others consider these tools as facilitators for students' content knowledge construction (Ozgun-Koca, Meagher & Edwards, 2011). The same belief might be held for teachers concerning their own content knowledge development. In a sense, teachers decide whether technology should follow content, content should follow technology, or both for the construction of either students' or their content knowledge. Ozgun-Koca, Meagher and Edwards (2011) found teachers' experiences with technology and teaching practice do little to change beliefs favoring learning on paper prior to an interaction with technology. According to the teachers, content should be taught first, and then technology is used to practice the mathematical content knowledge. Such beliefs could also hinder teachers' development of content knowledge through the use of technology.

In order to support teachers' development of mathematical content knowledge through technology, teacher education programs need to offer mathematics courses in which students are given opportunity to learn mathematical concepts, multiple representations and relationships among these different representations through frequent use of technologies as DGS, graphing calculators and spreadsheets. Such experiences could enable these teachers to recognize both limitations and capabilities of a certain technology before teaching with this technology, and how these limitations and capabilities might be turned into benefits for students' conceptual understanding. For example, the capacity of a calculator giving an error message at some situations might create chances for making sense of mathematical concepts and what is behind the error (Doerr & Zangor, 2000).

In regard to the literature reviewed in this sub-section about technology, teacher knowledge for 21st century needs, and beliefs about technology, there have been empirical efforts to examine teachers' knowledge development when technology was used, and others have investigated teachers' beliefs about mathematics, about teaching mathematics, about teaching mathematics with technology, the links among these beliefs and their influence on teacher knowledge development. However, there are limited studies investigating such links and the nature of teacher knowledge and beliefs when technology is used as a cognitive tool for the preparation of mathematics teachers within a mathematics content course.

Chapter 3 outlines my use of the literature and the methodology I intend to use to answer my research questions. I present my theoretical framework, and describe the details of my methodology including participant selection, research setting, instruments, data collection and analysis techniques.

CHAPTER THREE – METHODOLOGY

Context

My study examined PSTs enrolled in a technology-focused geometry course seeking a Master's of Arts in Teaching at a Southeastern research university. The Master of Arts in Teaching (MAT) in middle level education is an accredited graduate degree program, which provides individuals who have received a bachelor's degree in another field an opportunity to transition to a teaching career in middle level education. PSTs within the Middle Grades MAT program complete 36 hours of graduate work within educational theories, subject-matter knowledge, educational research methodology, and field experiences. Upon completion, PSTs will be certified to teach grades 5 to 8 in a specific subject, or may also dual-enroll in two subject areas (English, Social Studies, Mathematics or Science).

The program curriculum includes pedagogy-based courses on educational psychology, curriculum, human growth and development, educational tests and measurement, reading, and addressing the needs of diverse learners. Two field-experience courses (i.e. practicum and student teaching) aim to provide students with teaching and practice within their selected content area. Finally, the program requires students to complete 12 hours of content within their subject area. PSTs seeking to be middle school mathematics teachers take a 4-course mathematics content sequence consisting of Number & Operations, Algebra, Geometry, and Probability & Statistics.

MTHS 7090 – Geometry for the Middle Grades, is the geometry course in the sequence, which served as my main research site. I selected this course as my research site because it is a mathematics content course whose participants are pre-service teachers (PSTs) seeking an initial teaching certificate. Secondly, one of the learning outcomes expected from PSTs enrolled in this course is to use dynamic geometric software flexibly and fluidly during problem-solving tasks. Dynamic geometry software used for problem solving by PSTs allowed me to examine their Specialized Content Knowledge (SCK) development while technology was used as a cognitive tool. There were alternative research sites to this geometry course at this university, such as Technology in Secondary Mathematics, or Instructional Technology Strategies. The former alternative option was a teaching methods course that emphasized strategies to use technology for secondary mathematics instruction, the latter option was a technology methods course offered to secondary mathematics education majors. I did not select either of these two options because these courses are methods courses, not mathematics content courses given by the department of mathematics. I focused on finding a research site with a priority of teaching/learning mathematics content rather than pedagogy because my research agenda for this study was on the investigation of technology usage for the development of a type of subject-matter knowledge, not pedagogical content knowledge. Regarding this reason, MTHS 7090 was a better choice to collect this type of data.

MTHS 7090 focuses on understanding fundamental geometry topics pertaining to the middle grades curriculum through instructional methods such as learning by doing and constructing mathematical knowledge cooperatively. Learning goals for this course include: describing and understanding geometry content related to middle grades; connecting content to the Common Core State Standards for Mathematics; expertise in using dynamic geometry software for problem solving; creating inquiry-based geometry tasks to be used in middle school classrooms; and using and connecting multiple representations for geometry concepts (Appendix G for *the course syllabus*). As a requirement of the course, students are expected to download the Geometer's Sketchpad (GSP) onto their personal computers, and buy a textbook called "*Exploring Plane and Solid Geometry in Grades 6-8 with The Geometer's Sketchpad*" (Bennett, 2011). Throughout the semester, there were thirteen class meetings.

The instructor of the course assessed students on their performance in pre-determined assignments such as Individual GSP Labs, GSP Group Investigations, Student Geometry Portfolio, a midterm and a final exam. At the end of the first class, the instructor assigned PSTs to use GSP by themselves on their PC as an introduction. For the next class meeting, she also assigned an Individual GSP Lab that focused on the construction of a perpendicular bisector for a given line segment. PSTs were expected to follow the steps on a handout for the given Individual GSP Lab and answer questions in order to understand the geometry content embedded. The steps in the handout were straightforward and easy to follow for new users who did not have any experience with the software before. The second class meeting started with the review of this assignment as a whole class activity. PSTs were getting used to the software with guidance of the instructor. In addition to Individual GSP Labs, the instructor encouraged PSTs to use GSP as a tool to solve geometry problems assigned during class meetings; and she modified some assignments in order for PSTs to use GSP more at home.

During the planning process of my research, I was in contact with the instructor of the course in order to obtain permission for access to her classroom. Prior to the data collection in fall 2013, I interviewed her to understand what a day of typical instruction would look like and what her goals for PSTs enrolled in the geometry course were. Through this interview, I also talked about the details of when to conduct interviews regarding her content flow throughout the semester, and which class meetings would be better for me to observe for my research agenda. After this initial interview, I decided to observe each class meeting, and to interview the instructor briefly right after each class meeting so as to discuss the class that I observed and have a better picture of the course as the unit of analysis of my first two research questions in my study.

Regarding these interviews, the instructor emphasized the importance of inquiry-based teaching and learning, reflection, metacognition, assessment of PSTs' previous knowledge and their motivation for teaching and learning mathematics. The majority of her classroom activities were open to PSTs' different interpretations, analysis and conduct. Her typical instruction started with the presentation of an open-ended problem and a small whole-class discussion to engage PSTs. After that, she allowed PSTs to form a group to work on the presented problem collaboratively and to explore the mathematics within the problem. The final part of a typical instruction included an explanation phase by which PSTs presented their methods for the problem and discussed with others. The instructor deliberately chose these groups whose solutions involved errors in order to allow others to be aware of possible misconceptions their students might encounter in the future. The instructor also used exit tickets at the end of almost each class meeting to help

PSTs reflect on the mathematical focus of the class meeting, its connection to their previous knowledge, and its relationship to the real world. The instructor was also reflective on her teaching methods. During one of the class meetings, she asked PSTs to evaluate her instruction in terms of content, assessment techniques, and teaching methods. She stated during one of the interviews after the class meeting that she also requested their evaluation to help them be aware of teaching processes embedded within her instruction.

The research data were collected during the implementation of this graduate course in fall 2013 semester. Students and the instructor met for three hours weekly, for a total of 13 weeks during the semester.

Researcher's Role

In this study, as the researcher, I investigated the mechanism occurring within the course class meetings between the instructor and PSTs. In addition, I examined PSTs' knowledge development and how their beliefs about mathematics, teaching and technology influence their knowledge development. As I was not the instructor for the course I studied, the participants for this study only viewed me as a researcher. I employed observations as a data collection method and engaged in non-participant observation in order to capture the process of knowledge development without actively influencing it.

I am not a native speaker of English, but have had some undergraduate teaching experience in the US. Having an international background brought both opportunities and challenges for the quality of the research. Using English as a second language was a challenge for the data collection and analysis. For the improvement of the quality of the data collection and analysis, the chair of the dissertation committee supported my understanding of the events occurred during class observations and interviews. Moreover, having an international background also provided different lenses and perspective for data analysis. Because I have not been exposed to the US schooling culture for so long, I considered I recognize different phenomena that might not be recognized by American people. The recognition of these kinds of happenings might be difficult for people who are living in the same environment for a very long time.

Participants

The instructor of the course is an assistant professor in the same university the study took place. She has been holding a joint appointment in the departments of mathematical sciences and teacher education since August 2012. She has her bachelor and master degrees in mathematical sciences, and her PhD degree in learning sciences. Before her graduate studies, she had been teaching secondary mathematics in urban settings for six years. As mentioned, her teaching philosophy emphasized social constructivism and inquiry-based instruction. Her interviews after class observation indicated she favored PSTs to explore mathematical facts, terms and theorems collaboratively, and then to explain their findings to the whole class. She also viewed technology, especially GSP, as an opportunity for their exploration of geometrical phenomena and to understand geometry conceptually. When I was collecting data for this study, she was also the instructor of the methods course offered to pre-service middle grade mathematics teachers. In other words, a subset of PSTs participating in this study were taking both the methods and the geometry course.

PSTs participating in this study were graduate students seeking teaching certification for middle grade mathematics education. They have bachelor degrees from different departments such as psychology, religion, business administration, economics, marketing, financial management, communication, electrical and computer engineering, nursing, physical sciences or engineering. MAT in middle level education is a two-year program. Accepted graduate students, as mentioned before, are required to take four content courses. PSTs enrolled in MTHS 7090 – Geometry for the Middle Grades Course were in their first or second year of their graduate program. Based on current cohort enrollment, 16 PSTs enrolled in this course. 11 out of 16 PSTs were also concurrently enrolled in their mathematics methods course for middle grades. All PSTs enrolled in the geometry course were invited to participate in the study at the start of the fall semester, and volunteered to be a part in the study by signing their consent forms (N=16).

Participants enrolled in the geometry course did not have instructional technology experience and knowledge at the beginning of the semester. They also had not taken any geometry courses or reviewed geometry content since high school. Out of 16 participants, there was only one participant who had teaching experience (computer science, physical education and science) before the study, the rest of the participants did not have teaching experience.

Prior to this geometry course, a subset of these PSTs took an algebra course for middle grade mathematics education in summer 2013. The instructor of this course, who was different from the instructor of the geometry course, emphasized the use of manipulatives as alternatives methods to understand basic facts, procedures and theorems in algebra. Rather than giving a common procedure, PSTs in this course were guided to find procedures by themselves through interaction with other PSTs.

Not all data instruments were administered on all participants of the study. For example, only those who attended both teaching methods and geometry course (N=11) in this semester received the Mathematical Knowledge for Teaching (MKT) test. I decided to collect this data only from one portion of the participants because the instructor did not want to allocate a lot of time spent on research rather than instruction. Considering that the MKT test takes one hour to complete, the instructor and I collected this data during the teaching methods class hour.

Furthermore, not all participants were interviewed throughout the semester. An entrance survey and the MKT assessment were administered at the beginning of the semester to be able to identify participants' MKT level and their beliefs about mathematics, teaching and technology. A knowledge and belief profile for each participant was created with respect to the preliminary results from the MKT test and entrance survey. Regarding varieties in their SCK level in the MKT test and responses to belief-related questions at the entrance survey, I chose six participants to follow for interviews throughout the semester.

Overall, with respect to the data collection procedures pursued at the beginning of the semester, data from different participants were utilized in answering each research question (Table 3.1). Participants whose data was used for the third research question were nested in the participants whose data was used for the first research question, which were nested in the participants whose data was used for the second research question. Different research focuses and methodologies chosen for each research question determined the sampling method, as convenience sampling was used for the first two research questions, and judgment sampling for the third research question (Marshall, 1996).

	Research Question 1: SCK Development	**Research Question 2: TCK Development**	**Research Question 3: Belief Change**
Participants	11 Participants enrolled in both geometry and teaching methods course	16 Participants enrolled in the geometry course	6 Focal Participants enrolled in both geometry and teaching methods course
Sampling Method	Convenience Sampling	Convenience Sampling	Judgment Sampling
Instruments administered	All Instruments administered *except Interviews*	All Instruments administered *except Interviews and MKT Tests*	All Instruments administered

Table 3.1: Participants and Sampling Method for each Research Question

All 16 PSTs completed the entrance survey at the beginning of the semester. I analyzed these data in order to make sense of participants' initial beliefs about mathematics, teaching and technology. Three PSTs' responses to belief-related questions were not clear for categorization. Because of this reason, I excluded these three PSTs for the selection of focal participants. However, as they appeared in the list below (Table 3.2), they were still participants within the study.

The rest of the PSTs' responses to the belief question about mathematics was categorized into two (*instrumental* versus *platonic* view of mathematics) regarding Ernest's framework (1989). None of the participants' responses for this question showed evidence of the view of mathematics emphasizing mathematics as a human construction and invention. For these 13 PSTs, I then used Kuhl and Ball's framework (1986, cited in Thompson, 1992) and categorized their responses to the belief question about teaching into three (*learner-focused*, *content-focused with an emphasis on conceptual understanding* and *classroom-focused*). PSTs' responses did not show evidence of the teaching belief that can be categorized as content-focused with an emphasis on performance. Finally, I categorized these PSTs' responses to the belief-related question about technology. I used Chen's framework (2011) and labeled their responses as *instrumental* and/or *substantive*. Some PSTs' responses showed evidence for both of the categories, which were labeled as *both*. Table 3.2 presents findings from this preliminary analysis for belief categorization:

Participants[1]	Math Beliefs	Teaching Beliefs	Technology Beliefs
Kristin	Instrumental	Classroom-focused	Substantive
Derek	Instrumental	Classroom-focused	Both
Cameron	Instrumental	Content-focused emphasizing understanding	Both
Victor	Instrumental	Content-focused emphasizing understanding	Instrumental
Kathleen	Instrumental	Learner-focused	Instrumental
Leonard	Instrumental	Learner-focused	Substantive
Richard	Platonic	Classroom-focused	Instrumental
Katherine	Platonic	Classroom-focused	Instrumental
Cindy	Platonic	Classroom-focused	Both
Karl	Platonic	Content-focused emphasizing understanding	Substantive
Abby	Platonic	Content-focused emphasizing understanding	Both
Erica	Platonic	Learner-focused	Both
Samuel	Platonic	Learner-focused	Substantive
Kaci	Undetermined	Undetermined	Undetermined
Laura	Undetermined	Undetermined	Undetermined
Jasmine	Undetermined	Undetermined	Undetermined

Table 3.2: Participants' Preliminary Belief Profiles at the Entrance Survey

Accounting for variation in responses to belief-related questions in the entrance survey (Table 3.2), I selected six focal participants (Kristin, Cameron, Kathleen, Richard, Karl and Erica).

Brief Description of Six Participants Interviewed

For the six participants interviewed, Figure 3.1 represents z-scores for their performance from the MKT pre-test where the sample included 16 PSTs participated in

[1] Participants' names are pseudonyms.

this study. In this diagram, Kathleen, Karl and Erica seemed to have performance over average for the overall MKT and SCK sub-domain, where Kristin, Richard and Cameron are below the average.

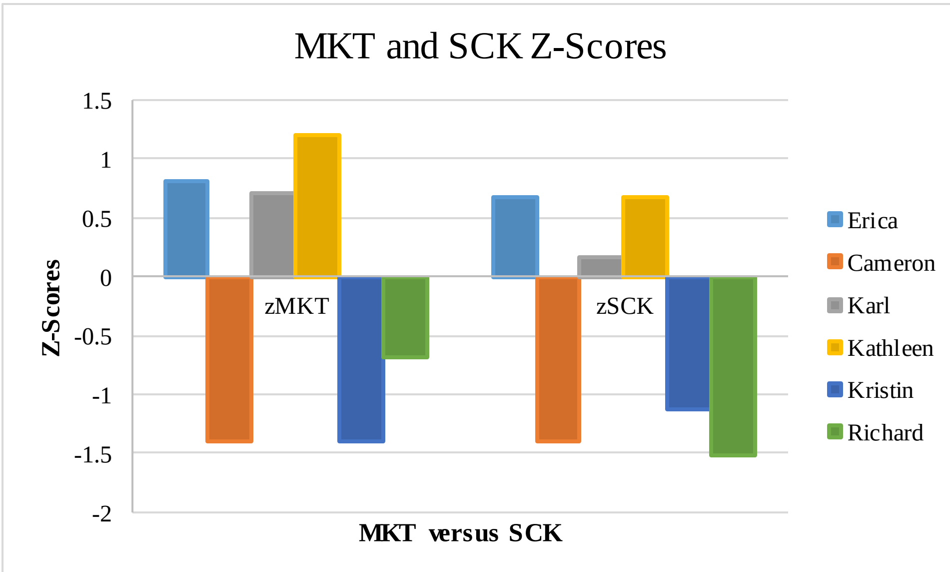

Figure 3.1: MKT Pre-Test Findings (Z-Scores)

In the following sub-sections, I describe the background of each of six participants that were interviewed. I mainly used information from their entrance survey to present an account of their previous professional background. However, I also utilized data from their first interviews in cases where the data in the entrance survey were not enough to describe their beliefs at the beginning of the study.

Kathleen.

Kathleen was 35 years of age and came to the teaching profession from her career as a hair stylist. Before enrolling in the MAT program, she had professional experience in several areas such as management, barbering, cosmetics and interior design. She was a hairdresser for the last 12 years. When asked whether she recalled her geometry knowledge from high school years, she answered that mathematics, including geometry, was a part of her prior job: "There is a lot of angle involved in that when you're cutting hair. Actually the way that you hold it out from the head and how you cut the hair depends, determines how it's going to lay or move." At the beginning of the study, she stated in the entrance survey that, "mathematics is a subject consisting of foundational laws, which is objective; but the application of laws is subjective." In addition, she viewed mathematics more objective compared to other disciplines. She explained how the meaning of a poem could be subjective as it can depend on who has read it. But, she considered mathematics as objective because of its accuracy and certainty. In the entrance survey, Kathleen answered the question about how to teach mathematics by referring to the construction of students' knowledge built upon their prior knowledge through inquiry methods. I considered this statement as she valued constructivism for her teaching, which allowed me to categorize her first belief about teaching with learner-focused. Even though Kathleen did not have a lot of experience with instructional technologies, her survey results suggested she viewed technology as something to be used for saving time during instruction.

Karl.

Karl was 21 years of age and completed his undergraduate major in economics. In the entrance survey, Karl viewed mathematics as a certain discipline independent of opinions, which requires critical and logical thinking. He also referred to this notion during the first interview by saying that "mathematics is a way of thinking logically and analytically … to apply it to [real world] situations". He also stated that mathematics cannot be attained simply by knowing all definitions, facts, procedures and related terms; understanding mathematics requires knowledge beyond compartmentalized mathematical entities. In the entrance survey, Karl considered that teaching mathematics should first focus on understanding important concepts; and then the instruction should move into problem solving for the application of concepts. These concerns were reflected within his choice of statements to demonstrate the content-focused view with understanding as his primary belief about teaching. At the beginning of the study, Karl considered technology as a support for instruction, but not a replacement for it, which was closer to the substantive belief about technology.

Richard.

Richard was 43 years of age and who had previous work experience in business. He was the only participant interviewed that had teaching experience, not in mathematics, but physical education, science and computer science. Even though he taught computer science, he also stated that his instructional technology knowledge was limited. When his beliefs about mathematics was asked in the entrance survey, he identified mathematics as a certain subject with one correct answers, but through a variety of methods. His beliefs about mathematics also indicated a language like image that was common among different cultures. To elaborate on this opinion, Richard considered that mathematical premises, constructs and understanding would be the same everywhere over the world. Even though Richard had teaching experience, he did not state anything about his teaching beliefs considering that he did not have required knowledge for this part in the entrance survey. Because of this, I looked at his first interview as his preliminary belief about teaching mathematics. His first interview showed that he signified both rote memorization and discovery as instructional techniques. He stated rote memorization is necessary for the acquisition of definitions and well-known procedures in solving mathematical problems. However, he also mentioned that students should be given discovery opportunities to see the relationships among mathematical concepts. From the given statements, he prioritized a classroom-focused view of teaching. In the entrance survey, Richard also thought that technology should be integrated into instruction because of the needs of the society today, but with an emphasis on understanding while using it.

Kristin.

Kristin was a 53 year old female who had worked as a nurse for many years before choosing teaching mathematics as a new career. The same as the rest of the interviewees, she did not have any instructional technology experience in general or specific to dynamic geometry software. Regarding this, she stated she was not very comfortable with technologies, and preferred to do things in old way rather than with technology. At the beginning of the study, she viewed technology as a tool to make life easier. She gave a couple of examples from daily life to show she used technology in easing tasks, such as using GPS instead of using a paper map or using cell phones for communication. However, these examples showed how she relied on their accurateness without any doubt. Similar to Karl’s beliefs, in the entrance survey, Kristin thought mathematics required critical thinking. In her entrance survey, she referred to her children and how they were struggling with mathematics in school. Because of such experience with her children, she stated that she would make an attempt to change this point of view for struggling kids during her teaching. In addition to that, in the entrance survey, Kristin viewed mathematics as an objective discipline consisting of algorithms that can be applied in real life. Similar to Richard, Kristin's teaching beliefs signified planning, classroom management and assessment in the entrance survey. She also thought that the introduction part of the lesson might focus on students' engagement through real life application.

Erica.

Erica was a 21-year-old female who majored in business administration before she decided to teach mathematics. Similar to Karl's beliefs about mathematics, she viewed mathematics as something that required conceptual understanding and problem solving. Departing from that, her initial teaching beliefs emphasized the importance of students' reasoning and understanding. In the entrance survey, she stated "technology could be beneficial if it can be used properly." In other words, she was aware that technology required expertise and instructional skills for its effectiveness in classroom. Her response showed both instrumental and substantive beliefs about technology at the beginning of the study. Erica gave an example from her teaching internship to illustrate her beliefs about teaching. In this example, her cooperating teacher was overemphasizing keep-change-switch procedure in subtracting two negative integers. Erica found it quite incomplete without showing the reason for this procedure. She proceeded to demonstrate the model on a number line to help students make more sense of the procedure mentioned. Out of six participants interviewed, Erica was one of the teachers who prioritized a learner-focused teaching view.

Cameron.

Cameron was a 28 year old male who had majored in business and worked at a pharmacy company for couple of years. His entrance survey did not include rich information about his beliefs about mathematics. He only stated that mathematics was used through our lives. Cameron considered the meaning of mathematics depends on what area of mathematics he is looking at. In general, he described mathematics as the art of expressing ideas numerically, but added to that by referring to geometry as expressing shapes in terms of angles, measures and constructions in order to make them exact. Cameron was the other participant who primarily selected statements representing the instrumental view of mathematics (Ernest, 1989). As an explanation to his choice of statements, he considered that basics such as definitions, terms, procedures and facts were the foundation of mathematics; the relationship among them would come next in order to be good at mathematics. His teaching beliefs were also limited into his thoughts about the priority of mathematics topics rather than how to teach them. He considered algebra and basic mathematics were first to be taught prior to geometry and calculus. He thought algebra and arithmetic are easier to find in everyday life for students compared to calculus and geometry. In addition to that, his beliefs about teaching also emphasized the use of dynamic teaching strategies to reach multiple learning styles. Because he mentioned content specifically in his response, his preliminary teaching belief was considered as content-focused emphasizing conceptual understanding. Regarding his technology-related beliefs, his response in the entrance survey indicated he viewed technology as a means to increase students' engagement and students' learning. According to Cameron, instructional technologies such as GSP provided precision, and additional opportunity to make sense of the content if they did not understand first on paper. However, he also mentioned that such opportunity can manifest depending on the students' background. For example, he considered that technologies might hinder some students' learning if they only use them to measure or to plug numbers in order to attain an answer or representation. Regarding his statement of pros and cons of technologies, he indicated a balanced state between substantive and instrumental views of technologies.

The first and third manuscripts focus on findings from a sub group of my focal participants: Kristin, Kathleen, Richard, Karl, and Cameron. The second manuscript presents results based on data from all 16 participants who were present during each class meeting.

Methodology

I used a single case study with embedded units design as the methodology for the examination of the second research questions, and used a holistic multiple-case study design for the first and third research questions (Yin, 2008). For the second research question, while the geometry course itself with its participants such as PSTs and the instructor of the course was the case of analysis, PSTs in this class and its subset that were interviewed became the embedded units.

The study can be considered to be a multiple case study for the first and third question as there was a comparison among knowledge level and belief profiles of participants that were interviewed. For these research questions, personal histories, experiences, the level of their SCK and/or TCK, and beliefs about mathematics, teaching and technology of each participant interviewed were examined as the phenomenon within the context; and all this information was developed into a case for each participant. Moreover, the theoretical propositions for knowledge development and its connection to beliefs necessitated the examination of phenomenon in a multiple-case study design because of the emergence of possible subgroups with respect to different levels of SCK as well as different belief profiles (Yin, 2008). As described, each participant who was interviewed created his/her context regarding his/her level of knowledge and different type of beliefs about mathematics, teaching and technology. In other words, each participant according to his/her different level of knowledge and beliefs created a case for a multiple comparison.

I selected the multiple-case study as my research methodology for the first and third research questions in order to replicate cases for possible theoretical generalizations. Case studies are generalizable to theoretical propositions, but not to populations. They do not deal with a sample like in an experiment. The goal of generalization in case studies is not *statistical* generalization, but rather the generalization of theories, which is called *analytical* generalization (Yin, 2008). A theory can emerge even from a single case under investigation, but this theory has to be replicated with multiple cases to administer replication logic. Through the use of replication logic, the external validity of the case study would also increase (Yin, 2008).

Even though I investigated knowledge development and causal links between the knowledge development and teachers' beliefs, this study's main purpose was not to form statistical interferences, but to understand the phenomenon of knowledge development for teachers while immersed within a particular technology. While examining such a complex phenomenon, I hypothesized teachers' beliefs and experiences also influenced the direction of their knowledge development. In order to examine this, one would design and conduct a causal comparative study (Ary, Jacobs, Razavieh, & Sorensen, 2010). However, this hypothesis was not to be investigated through experimentation. Rather, I was more interested in the process and the reason for the process of knowledge development. In this respect, a qualitative study, more specifically a case study approach, was the most appropriate methodology to examine factors and their relationships of real-life phenomenon, within a specific context.

Data Collection

The data collection process coincided with the beginning of the MTHS 7090 course in fall 2013. With the permission of the instructor of the geometry course, the aim and purposes of the project were introduced during the first class meeting of the course. PSTs were also informed as to privacy and confidentiality issues for the project. After that, PSTs were invited to participate in the study. Informed consent forms were collected from all PSTs who volunteered to be participants.

Four types of data were collected during the semester: 1) surveys and instruments to measure beliefs and knowledge; 2) course artifacts, 3) participant interviews, and 4) non-participant observations (Yin, 2008). The collection of different types of data, such as tests, surveys, documents, interviews and observations, allowed me to triangulate the data through multiple sources of evidence. This also created a chain of evidence, and trustworthiness for the findings from data.

Surveys and Instruments

Throughout the study, there were two surveys. A survey was administered at the beginning and at the end of the study. During the first class meeting of the geometry course, after PSTs signed their consent forms to volunteer in this study, they were asked to fill out the *Entrance Survey* (Appendix A) that was designed to collect information about their background and establish their current beliefs about mathematics, teaching, and technology, and their Technological Pedagogical Content Knowledge (TPACK).

The data from this survey were used to inform my selection of focal participants followed to be interviewed to be embedded units for the second research question, and to be cases for the first and third research questions. This survey included a part which was adapted from a survey designed by Schmidt and his colleagues (2009) to assess PSTs' TPACK. The original survey has 57 Likert-scale type items that assess teachers' content knowledge, pedagogical knowledge, technology knowledge, pedagogical content knowledge, technological content knowledge (TCK), technological pedagogical knowledge (TPK), technological pedagogical content knowledge, and models of TPACK for mathematics, social studies, science and literacy together. For each knowledge component, Schmidt and his colleagues documented reliability for each component at greater than 80%. In addition to these Likert-scale items, the original survey has nine open-ended questions examining participants' background, their previous experiences with technology and/or geometry concepts, and their beliefs about mathematics, teaching, and technology. For this study, I utilized the 57 Likert-scale type items from the original survey and modified both the demographic information questions and the-open ended questions at the end of the survey. The main purpose for the entrance survey was to create a beginning profile for participants' beliefs and their TPACK. The survey took approximately 30-40 minutes to complete. A parallel survey, the *Exit Survey* (Appendix B), was distributed to PSTs during one of the class meetings close to the end of the semester. I describe this survey as parallel to the entrance survey because it includes the same 57 Likert-scale type items but does not include the demographic questions. This survey also contains the same open-ended questions, but this time participants were asked to reflect about their experience in the course rather than their expectations from the course. In the Exit Survey, I focused more on participants' experiences, beliefs and knowledge development pertaining to the course they completed.

As mentioned, PSTs enrolled in both geometry and teaching methods course took the MKT assessment during their teaching methods class hour. Before that, the instructor sent them an email to guide them to open an account on an online server called LessonSketch, which is a practice-based professional development of secondary mathematics teachers and a forum for mathematics instruction. The website was used only to administer participants a pre-assessment of their Mathematical Knowledge for Teaching (MKT) for high school geometry.

The MKT assessment, designed by Herbst and Kosko (2012), includes a total of 28 items for Common Content Knowledge (CCK) (6 items), Specialized Content

Knowledge (SCK) (8 items), Knowledge of Content and Students (KCS) (7 items), and Knowledge of Content and Teaching (KCT) (7 items) separately. The internal reliability for these items is above 55% for each knowledge domain. According to Herbst and Kosko's description (2012), the items are dealing with definitions, properties and constructions of plane figures including triangles, quadrilaterals and circles, parallelism and perpendicularity, transformations, area and perimeter, three dimensional figures, surface area and volume, and coordinate geometry.

LessonSketch provided reports for each teacher as soon as the MKT test was completed. The administration of these tests provided an assessment of participants' MKT in relation to teaching high school geometry prior to the semester as well as at the end of the semester. The test developers provided the difficulty level of each item in IRT parameters. Using this information, each item was scored as 1-4. The MKT assessment was scored out of 99. Total scores for the sub-domains of CCK, SCK, KCS and KCT were calculated 27, 47, 13, and 12 respectively. With respect to these scores available, it is clear that the MKT test was heavily assessing SCK.

Moreover, test items were categorized according to the content they covered such as circles, geometric measurement and dimensions, quadrilaterals, congruency, triangles, transformations, angles, mathematical notation, and polygons. Each participant who took the MKT test also had a separate score for these content categories. I aimed to examine participants' development differentiated according to content categories as well as MKT sub-components from pre- to post-test.

The MKT post-test included the same items as in the MKT pre-test. Findings from the MKT post-test provided the final MKT level of participants and its comparison with the MKT pre-test demonstrated changes in participants' knowledge. Participants spent one hour to complete these tests.

Documentation and Course Artifacts

Course artifacts, such as the Syllabus of the Course, the Overview of the MAT Program, Individual GSP Labs and Assignments, Group GSP Investigations, Reflections and Exit Tickets, were collected. Prior to the analysis, participants' names were de-identified. While course artifacts were collected with respect to the negotiation with the instructor and the permission given by the volunteers for the study, some of these artifacts such as Individual GSP Labs and Group GSP Investigations were prioritized because of their richness in answering my research questions.

During the Individual GSP Labs, students were guided to complete textbook-based activities, which support their exploration of middle grades geometry concepts. After they completed these activities, PSTs were required to upload a GSP document as their final product of their exploratory and/or problem-solving work. Throughout the semester, the instructor assigned seven Individual GSP Labs as homework or in-class activities. These Individual GSP Labs covered content such as the construction of a perpendicular bisector, the construction of an equilateral triangle, properties of parallel lines, similarity, geometric transformations, tessellations, and spatial visualization for three-dimensional objects.

In addition to Individual GSP Labs, PSTs were expected to complete some GSP Group Investigations. For example, Shadow Data Gathering assignment asked PSTs to create a geometric model to demonstrate the relationship among the length of the shadow of an object, the height of a light source, the distance from the object to the light source, and the height of the object. PSTs as a group created a model on GSP and investigated the relationship with the help of technology, and sent the final version of their model to the instructor.

The instructor also asked PSTs to complete an assignment consisting of several GSP Lab activities as a replacement for the midterm. These labs covered content such as different types of similarities for triangles, angle types for parallel lines and transversal lines.

Interviews

A second data source was a series of semi-structured interviews, which were administered to the six focal participants. Throughout the semester, I conducted three interviews with each of the six participants.

These protocols and the geometry tasks embedded in them were field tested during the summer 2013. To do so, I asked two graduate students who had mathematics backgrounds to work on these geometry tasks. I also conducted preliminary analyses to observe the functionality of my analytical and theoretical frameworks for the actual study. Surveys and protocols were revised prior to the data collection in the fall.

The first interview (Appendix C for *the first interview protocol*) was conducted during the first month of the course and focused on the examination of participants' SCK and TCK. For this purpose, the semi-structured interview included a geometry task through which participants were guided to use GSP with a personal computer:

Square Construction

Susan claims she can construct a square by using the properties of a circle and lines having parallel or orthogonal properties. She began by constructing a circle by its center and around a point, and then constructing the radius (*see* Figure 3.2). Can you complete the construction? What do you think the student would do next? Predict their method based on what is described.

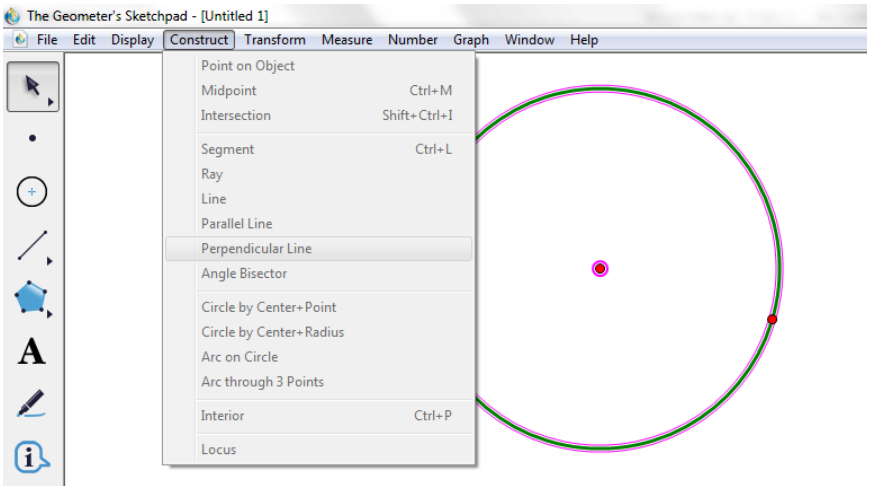

Figure 3.2: Screenshot from GSP Square Construction Task

The rest of the first interview included experience-based questions and belief-related questions. *Experience-based questions* were asked to clarify and understand the events that occurred during class observations. While some questions were asked to all participants interviewed, there were some questions asked specifically to one participant because she had a significant role within the event occurred during one of the class meetings.

Belief-related questions were in two forms. At first, I asked open-ended questions about participants' beliefs about mathematics, teaching and instructional technology. After, I asked more guided questions during which participants were recommended to read statements, to choose if they agreed, and to rank the ones they chose. These statements were created with respect to the theoretical frameworks that I elaborated in Chapter 2 for each type of belief separately. For example, there were separate statements representing instrumentalist, Platonist and problem solving views of mathematics; the choice of these statements and rankings by participants demonstrated their inclination towards a specific belief about mathematics in Ernest's framework (1989). The overall interview took 30-45 minutes.

The purpose for the second interview (Appendix D for *the second interview protocol*) was to capture any changes in teachers' SCK and to observe the role of technology usage in its development. It took 60-80 minutes to complete. The second interview was conducted during the 8^{th} week of the course and examined participants' experiences within the course so far, aspects of the content and pedagogy PSTs have learned, and what role technology played in their learning. In addition to these general experience-based questions, I also asked questions about the events that occurred during class observations.

This protocol also included a geometry task for participants to engage in:

The Triangle Inequality Theory

You taught that, in any triangle, one side has to be smaller than or equal to the sum of two other sides, and larger than or equal to the absolute value of the difference between the other two sides ($|a-b| \leq c \leq (a+b)$). A student using GSP states that a triangle having sides measured 2, 4, 5 inches cannot be formed (*see* Figure 3.3). However, regarding the triangle inequality, a triangle should be formed with these combinations. What should be the reason for the student's error?"

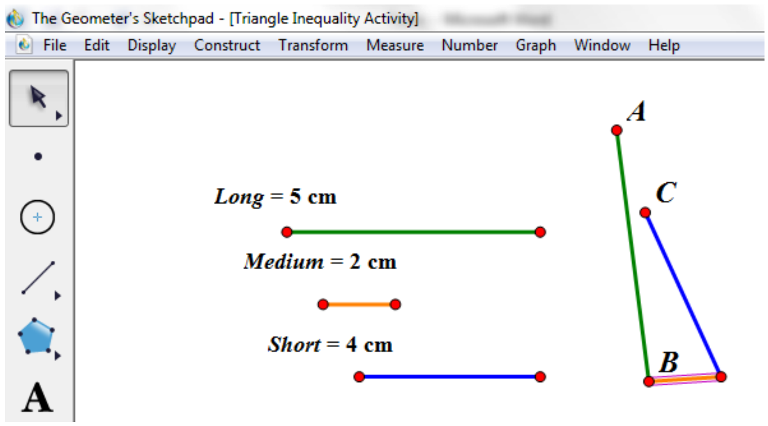

Figure 3.3: Screenshot from GSP Triangle Inequality Task

I designed the task to provide evidence of participants' SCK and TCK. In this task, I first asked participants whether they knew the Triangle Inequality Theory. After this introductory question, I presented a student's work on GSP, which indicated an error of the application of the Triangle Inequality Theory with given lengths of line segments of a triangle. GSP allowed the participants to dynamically adjust with the line segments to figure out the root of the student's error and the mathematics behind that error. If participants stated that they did not know the Triangle Inequality Theory, I guided them to use GSP in order to examine whether technology helped them understand the theory. The second interview protocol also included the same questions used in the first interview to examine participants' beliefs about mathematics, teaching and technology.

The third interview (Appendix E for *the third interview protocol*) was conducted at the end of the semester. It had a similar purpose as the second interview. With the third interview, I aimed to capture an end point for participants' gains and changes in terms of technology, pedagogy and content knowledge. The third protocol also included a geometry task designed to elicit evidence of participants' SCK and TCK:

Inscribed Circle of a Triangle

Regarding the animation within the GSP [*see* Figure 3.4a, 3.4b, 3.4c and 3.4d for snapshots from animation], what could be the relationship? How did you understand that? Could you explain it to me?

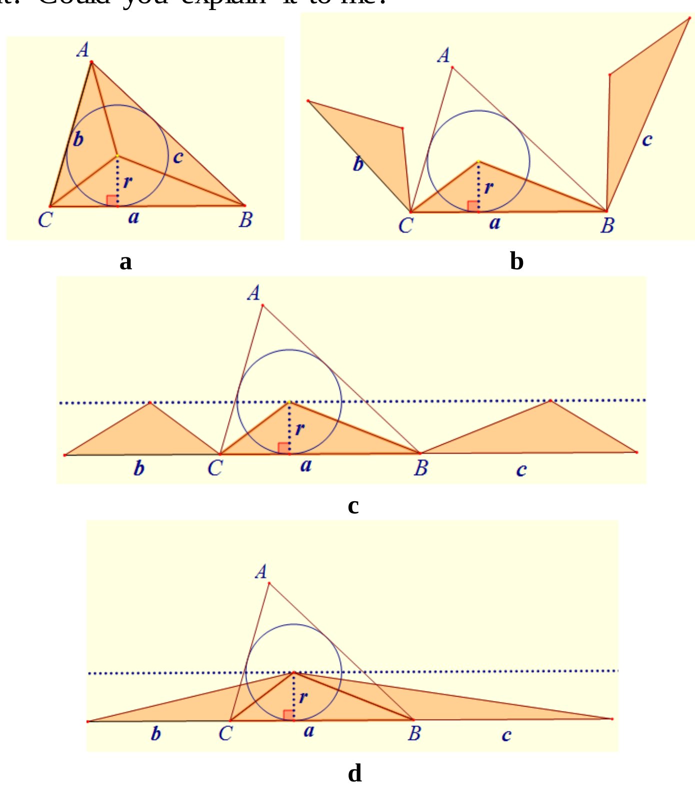

Figure 3.4: Screenshots from Inscribed Circle of a Triangle Animation

In this task, participants were asked whether they knew any relationship between the radius of a circle inscribed in a triangle, the perimeter and the area of the triangle. Depending on their response, participants were first asked to attempt to find the relationship using pencil and paper. Following this, four animations on GSP were presented as a guide. The four animations increased in levels of scaffolding depending on participants' knowledge and readiness. For example, if a participant recognized and understood the relationship from the first animation, I did not present the second animation.

Another geometry task during the third interview assessed participants' TCK and TK about geometric transformations on GSP, which was covered during one of the class meetings. In this task, participants were asked to reflect a point around x-axis, y-axis and x = y linear function, and rotate a triangle around a point.

The same as the first and second interviews, the third interview also included questions about participants' class experiences. For example, one of these questions examined their understanding of two of their classmates' work with geoboard during one of the class meetings. With this question, I aimed to look at participants' SCK about an unusual mathematical method used by their colleagues within an environment where technology was not used.

In addition, questions related to beliefs about mathematics, teaching, and technology are identical to ones used in the first and second interviews. I included the same questions addressing beliefs in all interviews to increase my ability to form a belief categorization for each participant coherently, and to see how their beliefs were changing throughout the semester. In other words, posing same questions three times triangulated the data, and strengthened evidence for each participant's beliefs about mathematics, teaching and technology.

I audio and video recorded every interview in order to capture the discussion as well as to document participants’ actions as they engaged in the geometry tasks. The interviews were transcribed verbatim.

Classroom Observation

The third data source was a series of non-participant observations (Dewalt & Dewalt, 2002). According to the negotiation with the instructor, I observed all class meetings from the beginning until the end of the semester. All observations except the first one were video recorded. In addition to the video recording, I took observation notes according to an observation protocol (Appendix F). These notes focused on major events and issues occurring for PSTs while they were interacting with GSP, with another material or manipulative, and with the geometry content in order to understand the topic or their colleagues’ strategies. More specifically, I tried to capture the essence of the class, the learning goals, the technology used, how the technology was used by PSTs and the instructor, which geometry topics and concepts were covered, whether there was any concept or procedure learned by PSTs, and what the role of technology was in this learning environment. Regarding the research questions, I sought to capture moments during which PSTs’ developed or used SCK or TCK pertaining to the geometry. Moreover, these observations were also used to guide my questions within the first, second and third participant interviews in order to make sense of their experiences in these class meetings.

After the observation of each class meeting, I created *write-ups* and *contact summaries* for each class observed. I generated write-ups, especially for the observation notes right after each class observed. Write-ups were the clear version of the collected data that included basic information with the contact. Considering the recommendations of Miles and Huberman (1994), the *contact summaries* of each class observation notes were created in order to focus on summarizing questions, to reflect on the experience, to guide the interviews, and to expand the existing interview protocols. While write-ups were descriptive in terms of what was observed, contact summaries were more interpretative and reflective. The contact summaries were used to answer the following questions:

- What were the main concepts, themes, issues, and questions emerging from this classroom observation experience?
- What were the hypotheses about the research questions?
- What should be asked to whom in the following interview?

The following table (Table 3.3) summarizes data sources for each research question and their connection with the manuscripts:

	Manuscript 1 **Research Question 1** **(*SCK Development*)**	**Manuscript 2** **Research Question 2** **(*TCK Development*)**	**Manuscript 3** **Research Question 3** **(*Beliefs and Knowledge Development*)**
Data Sources	MKT Pre- and Post- Tests Entrance and Exit Survey Interviews Field Notes	Entrance and Exit Survey Field Notes	Entrance and Exit Survey Interviews

Table 3.3: Research Questions and Data Resources

Data Analysis

Analysis of the MKT Tests

The data analysis process started after the administration of the MKT pre-test. The MKT pre-test was evaluated automatically through the LessonSketch website and scores were reported to me. These reports informed how each participant who took the test performed in the overall test as well as on its sub-constructs such as CCK, SCK, KCS, and KCT. In addition to that, reports also documented the difficulty level of each item as an IRT parameter. I used these parameters as a guide in determining the score for each item. After that, I calculated each participant's raw score, and converted them into a z-score. Furthermore, I formed a graph for each participant's z-score at the MKT pre- and post-tests.

Analysis of the Surveys

Participants' background information from the entrance survey such as their majors before they began MAT program in fall 2013, their work experience, their experience with geometry at college level, and their instructional technology experience was used to form the context of the study.

Furthermore, the TPACK survey part of the entrance and exit surveys was used to make sense of participants' TPACK at the beginning and at the end of the semester. For each TPACK component, a total score was created. For example, if a participant strongly agreed with all statements under the TCK Math component of the survey, her total score calculated as 5(Strongly Agree)*4(Number of Items under TCK Math) = 20. By using the same logic, scores for TPACK, TPK, TCK, TPACK for mathematics, and TCK for mathematics were calculated for each participant.

As mentioned before, the entrance survey provided a profile for each participant's TPACK and TCK. These profiles, in addition to SCK scores from the MKT test, were used in choosing participants to follow for interviews throughout the semester.

In order to see the change of TPACK for each participant, statistical analyses such as dependent t-test analysis between entrance and exit survey, calculating mean and standard deviation for TPACK total scores for entrance and exit surveys, and their graphic representations were conducted. I was aware while conducting dependent t-test analysis that the chance of having a significant difference was low. After all, there was not sufficient number of participants who took the survey. As a result, the distribution of the difference between participants' TPACK scores at the entrance and exit surveys was not even normally distributed.

Content Knowledge component of TPACK survey in the entrance and exit surveys provided participants' level of comfort with their mathematical knowledge compared to other contents such as literacy, science and social science. For all participants who completed the surveys, I looked at which content they were comfortable to teach at the beginning and at the end of the semester. It was expected participants felt more comfortable about their mathematical knowledge at the end of the semester compared to the beginning of the semester.

Participants' responses to open-ended questions in the entrance survey were first used for the selection of focal participants to be interviewed. By comparing and contrasting their profiles, I selected six participants to sample differentiation within participants' preliminary SCK and TCK levels.

Furthermore, open-ended questions in the entrance and exit surveys were coded. The coding process, which can be described as the division of the data collected into manageable chunks (Lodico, Spaulfing, & Voegtle, 2006), started with the data collection, which was a general situation for the analysis process in case studies (Yin, 2008). First, *descriptive codes*, or open codes (Corbin & Strauss, 1998), were generated on participants' responses in the entrance and exit surveys. Merriam (2009) describes *descriptive coding* as the process to identify unit of data that has potential to answer research questions. Because I was open to any possible information emerging from the data, I focused on taking notes and labels into the margins of the data sheet. At this phase, the labels were not necessarily be answering my research question directly. Labels for the descriptive codes were concretized more when I started to see repetition of the same or similar phenomenon within overall data. Next, I reexamined descriptive codes in order to make abstraction about big ideas. This final combination of codes and my interpretations allowed me to create pattern codes, or analytical codes as defined by Corbin and Strauss (1998).

After patterns codes were constructed, I matched those codes into the categories specific to the theoretical framework I used. For example, I used Ernest's framework (1989) to categorize participants' beliefs about mathematics. All pattern codes emerging from participants' responses to the question about their beliefs of mathematics in the entrance and exit surveys were matched with a category in Ernest's framework. I followed the matching process by relabeling participant's responses with categories such as instrumentalist view, Platonist view, or a problem solving view. Later, I created a table in which I presented the distribution of beliefs about mathematics and additional pattern codes among participants. The same procedure of open coding and category matching were used for the beliefs about teaching, and the beliefs about instructional technology with respect to the corresponding theoretical framework I used for each construct.

I had open-ended questions about participants' expectations from the course and their thoughts about GSP in the entrance survey. In the exit survey given at the end of the study, I asked participants to discuss how advantageous or disadvantageous it was to use GSP in learning geometry. These questions in the entrance and exit surveys examined participants' TCK. All participants' responses to these questions in the entrance and exit survey were coded descriptively, and then pattern codes were constructed. The distribution of these descriptive and pattern codes was represented in a table for the entrance and exit surveys.

After I had descriptive and pattern codes for every response and each participant in their entrance and exit surveys, I examined the distribution tables separately prepared for codes and beliefs emerging from the entrance and exit surveys in order to see the change from beginning to the end of the study. I utilized the different distribution tables for beliefs about mathematics, beliefs about teaching, beliefs about technology, as well as the codes for TCK.

Analysis of Interviews

The interviews included experience-based questions, task-based questions and belief-related questions. There were experienced-based questions specific to a participant, or general for each participant interviewed. Responses to experience-based questions were used to understand an event occurred in a class meeting in a better way. In other words, I used participants' responses to these questions in order to narrate important events pertaining to my research questions. While some of these questions were already determined to which research question they served, other questions emerged by going through the coding process defined above. This time, I used the theoretical frameworks related to CCK, SCK, and TCK. Each response was matched with a knowledge category; in some cases, there were more than one categorization for a response or part of the response. A statement was considered as CCK if it showed a definition or a usual procedure that answered a *what* or *how* question in mathematics. If the statement was answering a *why* question, demonstrating the mathematics behind a mathematical error or an unusual procedure, it was categorized as SCK. The statement was labeled as TCK if it was about the construction of geometrical objects, representations and links among representations within a technology-integrated environment. In the context of this study, GSP was the technology integrated. If there was no technology involved within the description of the event and in participants' responses, I automatically excluded TCK as a coding option.

Open-ended belief-related questions went through the same coding process: 1) constructing descriptive codes, 2) constructing pattern codes, 3) matching with pattern codes with theoretical categories. This time, I did not create distribution tables as the end product of the coding process because I only had six participants.

The other belief-related questions included statements to be chosen, ranked and discussed by participants. Their selection again provided evidence useful in classifying each participant's beliefs about mathematics, teaching, and technology. For each participant, I scored the statement they chose the most important with 7; the second important with 6; and I followed the same logic. The highest score given to a statement was determined depending on the number of statements available within the question. Because there were 7 statements available for the question about beliefs of technology, the first ranking statement chosen by the participant was scored with 7. As a result, each participant had an average score for each theoretical category. The highest score represented the primary beliefs about mathematics, teaching and technology. For each participant, I identified a primary and secondary belief about mathematics, teaching and technology during the first, second and third interviews. By doing that, I was able to examine the potential change in these beliefs throughout the semester.

The following example demonstrates how I scored participants' statements they chose and ranked from options. Kathleen chose the following statements to represent her beliefs about technology during the first interview:

1. Technology enables students to be more creative, interpretive and analytical.
2. Technology should mainly be used to increase the efficiency of the instruction.

3. Technology has potential to enhance students' learning and understanding.
4. Technology inhibits students to learn basic mathematical skills.
5. Technology can sometimes inhibit students' understanding of math if not used appropriately.

While 1^{st}, 3^{rd}, and 5^{th} statements represented an *instrumental* view of technology according to Chen (2011), the 2^{nd} and 4^{th} statements represented a *substantive* view of technology. That way, Kathleen had a substantive score equal to (7+5)/2=6, and an instrumental score equal to (6+4+3)/3 = 4.33. As a result, Kathleen's beliefs about technology during the first interview were labeled as primarily substantive, and secondarily instrumental.

For the task-based questions, I formed a rubric depending on the quality of the task. These tasks were used to measure participants' SCK and TCK with and without technology three times throughout the semester. The first interview included a task to complete a construction of a square on GSP. The following rubric (Table 3.4) was used for the square construction task in order to compare six participants interviewed. I examined whether the participant viewed a square as a quadrilateral having four equal sides, and each side was perpendicular to the sides next to them. I scored participants' construction of square on GSP depending on whether it was an approximation and how much participants used given information in the task.

Participants	Construction by compass & straightedge (Y/N)	Characteristics of square used during construction	Score for Student Procedure Prediction with GSP	Method used on GSP	Scaffolding used by the interviewer (Y/N)	Alternative method used on GSP to construct square (Y/N)	Method of alternative construction	Comparison of technology and paper
1								

Table 3.4: Rubric for the Square Construction Task during the 1st Interview

Each one of these tasks was labeled as *events* in the first manuscript in order to identify an example of SCK that was newly developed by participants, and what factors, including the use of GSP, influenced their SCK. The same tasks were analyzed in the same way for the third manuscript; additionally, participants' beliefs about mathematics, teaching and technology during three interviews were linked to their SCK demonstrations.

Analysis of Class Meetings

Field notes from each class meeting were used to determine the frequency of technology use for the overall semester, who was using the technology, which ways technology was used, and the distribution of teacher-led and student-led parts during observations. This information was identified in order to describe the context and class meetings in a better way.

By looking at the field notes taken during class meetings, I decided on important events and issues for my research agenda, watched these segments of videos, and transcribed discussion between PSTs and the instructor. The major purpose of these transcriptions was to narrate the events and issues. At this point, I also examined artifacts such as lesson materials, participants' class works and assignments to elaborate details of the events. After I narrated events within videos, I labeled them according to what type of knowledge they represented. Finally, I began the coding process on descriptions and transcriptions of events from class meetings pertaining to SCK and TCK.

With a group of pattern codes specific to each research question, I reexamined all data interviews and field notes. This process was followed by hypotheses construction for my research questions through the combination of pattern codes. Finally, I reexamined data to find confirming and disconfirming evidence for the hypotheses I constructed (Lodico, Spaulding, Voegtle, 2006).

The methodology for the third research question is a multiple-case study, which required an extended coding process. After pattern codes were found within each case from the coding process, a cross-case comparison was conducted to see commonalities and differences (Miles & Huberman, 1994). In general, during the analysis of the study, I proposed different explanations and varying possibilities for the events observed or data gathered (Yin, 2008). Each one of these analytical strategies strengthened the links between data and the claims; and increased the internal validity of the case study.

Coding the data with another graduate student in mathematics education provided inter-rater reliability for the study. This process also provided an investigator triangulation through different evaluators (Yin, 2008). Case analysis meetings were conducted for each two weeks so as to discuss preliminary findings and research directions both for data collection and data analysis.

In the next three chapters, I present three manuscripts with their different introductions, literature reviews and theoretical frameworks, methodologies, results and conclusions. Each manuscript represents my research for one of my three research questions.

CHAPTER FOUR – SPECIALIZED CONTENT KNOWLEDGE DEVELOPMENT

FOUR PRE-SERVICE MIDDLE GRADES MATHEMATICS TEACHERS' SPECIALIZED CONTENT KNOWLEDGE DEVELOPMENT WITHIN A TECHNOLOGY-ENHANCED GEOMETRY COURSE: A CASE STUDY

To be submitted to the *Journal of Mathematics Teacher Education*

Abstract

This study characterizes the development of Specialized Content Knowledge (SCK) in geometry within a technology-enhanced graduate content course for one semester. The research employed a multiple-case of four pre-service middle grade mathematics teachers. Both qualitative and quantitative data were collected, and factors affecting these two teachers' SCK development were compared and contrasted through their mutual experiences during the course and accompanying clinical interviews. The cross-case comparison indicated factors such as opportunity to justify ideas in geometry, the level of Common Content Knowledge, views about instructional technology as influential for these pre-service mathematics teachers' SCK development.

Keywords: Specialized Content Knowledge, Middle Grade Pre-Service Teachers, Dynamic Geometry Software

Introduction

In response to the changes in what and how teachers should be teaching mathematics within K-12 schools, various organizations within the mathematics education communities have proposed recommendations for teacher education programs. The quality, amount, and structure of mathematics courses for teacher preparation programs, instructional strategies, and content knowledge expectations from prospective teachers were highlighted within these recommendations (CBMS, 2001; Blair, 2006; NMAP; 2008).

Reports such as *Mathematical Education of Teachers* (MET) (CBMS, 2001), Beyond Crossroads (Blair, 2006), and *Foundations for Success* (NMAP, 2008) underlined the importance of the design of programs that support teachers to develop a *solid knowledge of mathematics* (CBMS, 2001). However, what constituted "solid knowledge of mathematics" has been interpreted by teacher educators quite differently. For many teacher educators, taking advanced mathematics courses and getting satisfactory grades have been considered as the means to develop this knowledge. This misinterpretation of solid knowledge of mathematics has been perceived as a challenge for the qualification of prospective mathematics teachers (CMBS, 2001).

The development of *Common Core State Standards* (CCSSM) for mathematics (CCSSI, 2010) has accelerated the need for teachers who have strong mathematics background with deep conceptual understanding (Porter, McMaken, Hwang & Yang, 2011). The *Mathematical Education for Teachers (MET) II* report (CBMS, 2012) indicated the same consideration with its coursework recommendations for middle school teachers as the previous reports. The document specified advanced mathematics courses for prospective middle school teachers to take for their professional preparation. These recommendations were asserted as an alignment of teacher education programs for the CCSSM (CBMS, 2012), and stressed the experience of advanced mathematics content through reasoning and sense making, so that prospective teachers could develop deep understanding of this content in relation to both elementary and high school mathematics concepts.

In this interpretation of deep understanding, prospective teachers know more than simply how to "do" middle school mathematics, and now also develop a special kind of mathematical content knowledge, germane specifically to the work of teaching mathematics, which is referred to by Ball and her colleagues (2001) as *Specialized Content Knowledge (SCK)*. SCK is a knowledge component of *Mathematical Knowledge for Teaching (MKT)* framework (Ball, Lubinski, & Mewborn, 2001; Ball, 2003; Ball, Thames, & Phelps, 2008). According to the MKT framework, mathematics teachers need to acquire and develop general mathematical content knowledge and a specialized one for teaching: SCK. SCK was defined as special mathematical content knowledge, which bridges well-known mathematical procedures with where they come from and why they were used in a certain way.

Conceptualization of SCK within the MKT framework served as evidence for the necessity of teacher education (Hill, Sleep, Lewis & Ball, 2007) and the importance of building knowledge for changes projected by reform-based standards (Goertz, 2010). This conceptualization also increased the challenge concerning how the development of such knowledge through teacher education programs would be accomplished. One possible way to accelerate teachers' development of mathematical content knowledge, emphasizing conceptual understanding and procedural proficiency, is the use of electronic technologies. Research has demonstrated that effective use of technology supports students' development of conceptual understanding (Mann, Shakeshaft, Becker, & Kottkamp, 1998; McCoy, 1996; Wiske, Franz & Breit, 2005; Roschelle, Shechtman, & Tatar, 2010). The same premise may hold true for the development of teachers' SCK when technology is used effectively as a tool to construct required mathematical content knowledge.

A narrowing of focus to literature addressing pre-service mathematics teachers' development of SCK for geometry content produced limited results (Speer & Wagner, 2009). The literature examining the development of SCK through the use of technology was also limited (Silverman & Clay, 2009; Silverman, 2012). Regarding the gap in the literature concerning the use of technology and the quality of geometry content courses for pre-service teachers' SCK development, I examined the experiences of four middle grade mathematics pre-service teachers and how these experiences impacted their SCK development process within a geometry content course. I assumed that technology would influence 1) the nature of common content knowledge of mathematics (Ball et al., 2008), and 2) the development of mathematical content knowledge specific to teaching. The study tested the extent of these assumptions. The following research questions guided my study:

1. How does a Technology Integrated Geometry Course influence pre-service mathematics teachers' development of SCK?
2. What is the role of technology in the development of pre-service mathematics teachers' SCK?

Theoretical Framework

Defining Knowledge as a Construct

Knowledge as a cognitive construct is one of the terms scholars have struggled to clearly define and describe. It is difficult to determine what to label as knowledge and how it is different from other cognitive constructs such as beliefs. Verloop, Driel and Meijer (2001) define knowledge "as an overarching, inclusive concept, summarizing a large variety of cognitions, from conscious and well-balanced opinions to unconscious and un-reflected intuitions" (p. 6).

Lemos (2007) lays out the discussion of whether information is correct or not in terms of its correspondence to the negotiated facts. According to Lemos (2007), there are three types of knowledge: 1) how to knowledge, 2) acquaintance knowledge, and 3) propositional knowledge, which is the knowledge of facts and true propositions. A proposition can be called true if and only if it corresponds to the facts. For example, the proposition of "four times two is equal to six" is a false proposition given by a pre-school child. The child might think that the statement is true, but it does not show that s/he knows the multiplication operation accurately because the statement does not correspond to the facts about multiplication. Whether a proposition is true or corresponding to the facts also determines the differentiation of knowledge from beliefs. Beliefs are related to values, attitudes and opinions, while knowledge consists of facts and true propositions (Pajares, 1992).

For the framing of this study, I defined knowledge as cognitive products that consist of procedural and conceptual propositions that might be projected onto facts negotiated by others as valid. For example, a person might propose an intuitive geometric claim. If this claim cannot be verified with facts in geometry that were negotiated by experts in the field, then I could not consider this claim as knowledge, but beliefs. Any statement in collected data was labeled as knowledge as long as it was used with certainty, and was linked to facts that could be considered the consensus of a group of professional people in the domain.

Mathematical Knowledge for Teaching

Though there have been efforts to reveal personal teacher knowledge through investigating teacher classroom behaviors and examining the propositions and facts behind their actions (Nespor & Barylske, 1991; Clandinin & Connely, 1996), research during the second half of the 1980s generated and described several new models and shared components of teacher knowledge (Leinhardt & Smith, 1985; Shulman, 1986). Generally, these models differentiated subject-matter knowledge from pedagogical knowledge which were treated as two separate constructs until the introduction of *Pedagogical Content Knowledge (PCK)* by Schulman in 1986. Shulman's (1986) model defined three kinds of content knowledge: 1) content knowledge, 2) pedagogical content knowledge (PCK), and 3) curricular knowledge. According to his description, subject-matter knowledge was composed of facts, concepts, and understanding of structures within a given subject. He described PCK as special knowledge, which helped teachers transfer what they knew as subject-matter to their instruction in a form that facilitated students' comprehension. After the introduction of PCK into the educational lexicon in 1986, several educational researchers in and outside of mathematics education investigated this construct within the classroom context and through this research, created new models of teacher knowledge (Carpenter, Fennema, Peterson, & Carey, 1988; Howey & Grossman, 1989; Grossman, 1990; Even, 1993).

While PCK was useful in investigating the relationship between content and pedagogical knowledge for the teaching profession, it could not answer all questions. Ball and Bass (2000) pointed out the need to bridge content and pedagogy more specifically for teaching mathematics. Researchers in the field of mathematics education (Ball, Lubienski, & Mewborn, 2001; Hiebert, Gallimore, & Stigler, 2002) criticized the focus of research on mathematics knowledge and its foundation from the examination of mathematics curricula. Rather than a traditional approach of looking at the content to configure mathematical knowledge needed for teaching, they claimed that classrooms should be the research sites in order to reveal the type of knowledge needed by mathematics teachers. These criticisms and the shift of emphasis in the field of education from formal to practical knowledge allowed Ball and her colleagues (Ball, Lubienski, & Mewborn, 2001; Ball, 2003; Ball, Thames, & Phelps, 2008) to develop and introduce the construct of *Mathematical Knowledge for Teaching* (MKT).

According to the MKT framework, mathematics knowledge for teaching was initially separated into subject-matter knowledge and pedagogical content knowledge.

These two facets were then further subdivided into three knowledge components. Figure 4.1 presents each of these components of the MKT Framework (Ball, Thames, Phelps, 2008). Because this study focused on SCK development, I carefully described SCK as a construct and how it could be differentiated from CCK.

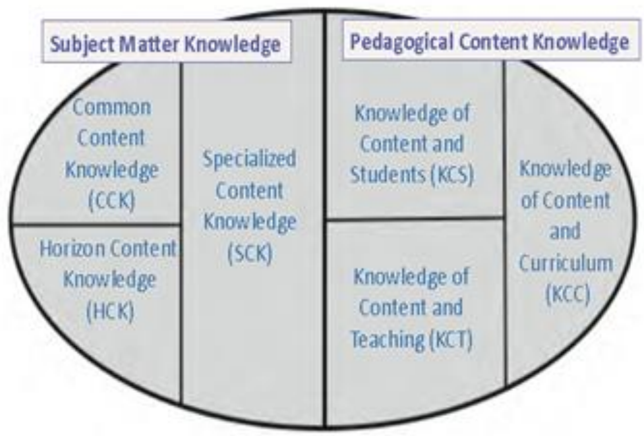

Figure 4.1: Representation of the MKT Framework (Ball, Thames, Phelps, 2008, p.5)

Common Content Knowledge (CCK) is defined as "knowledge of mathematics that was common across professions and available in the public domain" (Hill, Sleep, Lewis, & Ball, 2007, p. 131). The availability of CCK would enable teachers to make correct mathematical statements, master the content they teach, use terms precisely, differentiate an incorrect definition from the correct one, and recognize students' incorrect answers (Hill, Schilling, & Ball, 2004; Ball, Thames, & Phelps, 2008, p. 399). However, the authors underline that this mathematical knowledge is common to any profession which uses/applies mathematics.

In considering these descriptions, I defined CCK for this study as mathematical knowledge which an undergraduate student, who is not necessarily majoring in mathematics education, might develop through his/her tertiary study. CCK is factual, conceptual, procedural and algorithmic knowledge which enables teachers to recognize mathematical facts, procedures, strategies, define concepts correctly with mathematically acceptable terms and notations, and differentiate the correct answer from the incorrect ones for a given problem. For example, a secondary school mathematics teacher would know that $(x-h)^2 + (y-k)^2 = r^2$ is the algebraic representation for any circle where r denotes the length of its radius and (h, k) denotes its center. In addition, the teacher would also recognize that a circle on the Cartesian coordinate plane is not a function by applying the vertical line test. Both of these examples are a part of the teacher's CCK because the fact about the circle's algebraic representation and the procedure to determine whether a given shape is a function or not can be acquired, developed, and conceived by people in other professions as well.

Specialized Content Knowledge (SCK) is the knowledge of mathematics exclusively needed for the teaching profession (Ball, Hill, & Bass, 2005), which includes "building and examining alternative representations, providing representations, and evaluating unconventional student methods" (Hill, Schilling, & Ball, 2004, p. 17). The following list of teacher skills would describe what constitutes SCK and how it is different from CCK:

- Showing and representing mathematical terms and operations visually,
- Providing mathematical reasons for common procedures,
- Understanding mathematics behind students' unusual procedures and generalizing them if needed,
- Constructing real life problems related to specific mathematical concepts,

- Examining and unraveling the source of students' mathematical errors (Ball, Hill, & Bass, 2005; Hill, Sleep, Lewis, & Ball, 2007; Ball, Thames, & Phelps, 2008)

One of the main characteristics of SCK which differentiates it from CCK is it consists of knowledge used for contributing to students' learning, but not taught directly to students. That said a teacher would need to develop SCK, not in order to teach the information directly to students, but to utilize it when needed. In this respect, SCK is secondary to students' learning of mathematics during instruction. The teacher would focus on unpacking his/her CCK to design and achieve content-related learning goals and utilize his/her SCK to overcome possible student difficulties in understanding and making sense of these ideas. For example, when the teacher applied the vertical line test on a circle and stated that it is not a function, a student might ask why the vertical line identifies if a given graph is a function or not. Learning the reasoning behind the vertical line test might not be planned as a learning goal. However, a student might still ask such a question; and availability of SCK for the teacher about this concept helps him/her strengthen the student's understanding. Regarding this example, conceptual knowledge can be part of teachers' SCK as long as the teacher did not aim or plan to teach it to students.

Literature addressing SCK

Morris and her colleagues (2009) investigated the development of SCK during teachers' pre-service education. In this study, SCK was defined as knowledge of mathematics unique to teaching mathematics, and considered as necessary knowledge for teachers to develop skill in specifying and unpacking learning goals into sub-concepts. In this respect, the authors examined how pre-service elementary teachers unpacked learning goals into sub-concepts for planning, evaluation, teaching and learning. The authors found that pre-service teachers (PSTs) managed to identify sub-concepts for a learning goal in supportive contexts, but could not apply this knowledge for planning, evaluation, teaching and learning. Supportive contexts were the ones in which PSTs solved the problem by themselves, or examined students' incorrect responses.

While Morris and her colleagues (2009) studied pre-service teachers' SCK through clinical interviews, Bair and Rich (2011) examined the same phenomenon over the span of two mathematics content courses. The authors questioned why some teachers are better in unpacking their SCK while teaching than others. This exploratory study demonstrated that teachers who develop SCK organize their instructional activities with respect to simultaneous use of their SCK and PCK together. PSTs with lower SCK in this study only maintained the learning goal by posing follow-up problems that only include trivial changes to numbers. When these teachers were asked to create similar problems with non-trivial numerical changes, they could not maintain the level of difficulty. In addition, the study indicated PSTs who have insufficient CCK might still show development of SCK, but could not move to higher levels of SCK understanding.

Technology Influence on SCK Development

Computer tools create opportunities for users to interconnect mathematical topics in a dynamic and interactive way. These tools make the exploration of real life phenomena possible, allow learners to be exposed to central ideas, and create new mathematics to learn (Cuoco, Benson, Kerins, Sword, & Waterman, 2010; Fey, Hollenbeck, & Wray, 2010). The use of virtual manipulatives provides teachers an opportunity to experiment with geometry. For an objective of developing conceptual understanding of the area formula of a triangle, interacting with technology might enable users to question the validity of a theorem for different conditions (Hollenbeck, Wray, & Fey, 2010). Such theorems can be discovered through paper and pencil, but the use of dynamic geometry software enables a more time-efficient discovery. The dynamic functionality of these software allows users to observe how the area of a triangle is related to the area of a rectangle, and in which cases the area of the triangle is larger and why.

The literature examining the development of SCK through the use of technology was also limited. Silverman and his colleagues (Silverman & Clay, 2009; Silverman, 2012) studied the effect of an online collaborative environment on pre-service and in-service K-12 teachers' MKT within an online geometry and algebra content course. In the online collaboration environment, teachers privately posted their thinking processes concerning the solution of the problem. The instructor of the course orchestrated an online whole-class discussion around teachers' solutions and the instructional objectives. The study revealed the online discussions with whole class discussions allowed teachers to share their thoughts on teaching practices and pedagogical insights. Even though this study presented important findings regarding the characteristics of online collaboration that influence the development of teachers' MKT, it did not address how or which type of technology can trigger or limit this development. Furthermore, teachers' PCK more than their SCK was the focus of these studies.

Methods

Context

In this study, I examined PSTs enrolled in a technology-focused, graduate-level geometry course, Geometry for the Middle Grades, which took place at a Southeastern research university in the fall semester of 2013. There were 16 PSTs enrolled in the course and each was seeking a Master's of Arts in Teaching. The Master of Arts in Teaching (MAT) in middle level education is an accredited graduate degree program, which provides individuals who have received a bachelor's degree in another field an opportunity to transition to a teaching career in middle level education through initial certification. PSTs seeking to be middle school mathematics teachers take a 4-course mathematics content sequence consisting of Number & Operations, Algebra, Geometry, and Probability & Statistics. I selected the Geometry for the Middle Grades course as my research site as one of the expectations of the PSTs enrolled in this course was to use the dynamic geometric software, *The Geometer's SketchPad* (GSP) (Jackiw, 1995), flexibly and fluidly within the course's tasks. PSTs' use of GSP for problem solving allowed me a context in which to examine their SCK development.

The Geometry for the Middle Grades Course focused on understanding fundamental geometry topics pertaining to the middle grades curriculum through instructional methods such as learning by doing and constructing mathematical knowledge cooperatively. The learning goals for this course included: describing the geometry content of typical middle grades mathematics courses and identifying the core underlying mathematical ideas of these courses; describing the similarities and differences in course standards for middle grades geometry; using dynamic geometric software flexibly and fluidly during problem-solving tasks; using manipulatives during problem-solving tasks and identifying their appropriate use in secondary geometry classrooms and explaining, justifying, and writing proofs related to course content (Course Syllabus, 2013). As a requirement of the course, PSTs were expected to download GSP onto their personal computers. PSTs and the instructor met for three hours weekly, for a total of 13 weeks during the semester. The PSTs enrolled in this course had no experience with GSP at the beginning of the semester.

The instructor of the course emphasized the importance of inquiry-based teaching and learning, reflection, metacognition, assessment of PSTs' previous knowledge and their motivation for teaching and learning mathematics. The majority of her classroom activities were open to PSTs' different interpretations, analysis and conduct. Her typical instruction started with the presentation of an open-ended problem and a small whole-class discussion to engage PSTs. After that, she allowed PSTs to form a group to work on the presented problem collaboratively and to explore the mathematics within the problem. The final part of a typical instruction included an explanation phase by which PSTs presented their methods for the problem and discussed with others. The instructor deliberately chose these groups whose solutions involved errors in order to allow others to be aware of possible misconceptions their students might encounter in the future. She also used exit tickets at the end of almost each class meeting to help PSTs reflect on the mathematical focus of the class meeting, its connection to their previous knowledge, and its relationship to the real world. The instructor was also reflective on her teaching methods. During one of the class meetings, she asked PSTs to evaluate her instruction in terms of content, assessment techniques, and teaching methods. She stated during one of the interviews after the class meeting that she also requested their evaluation to help them be aware of teaching processes embedded within her instruction.

Data Collection

Three types of data were collected during the semester: 1) Surveys and instruments to measure beliefs and knowledge; 2) participant interviews, and 3) non-participant observations (Yin, 2008). The collection of different types of data, such as tests, surveys, documents, interviews and observations, allowed me to triangulate the data through multiple sources of evidence. This also created a chain of evidence and helped establish validity for my findings.

I created a survey, which was administered at the beginning of the study, consisting of nine open-ended questions examining participants' background, their previous experiences with technology and/or geometry concepts, and their beliefs about mathematics, teaching, and technology. To measure participants' SCK, I administered an online assessment of Mathematical Knowledge for Teaching Geometry (MKT-G) (Herbst & Kosko, 2012) at beginning and the end of the study. The assessment consisted of 28 items including items specific to assessing specific sub-domains of MKT: Common Content Knowledge (CCK) (6 items), Specialized Content Knowledge (SCK) (8 items), Knowledge of Content and Students (KCS) (7 items), and Knowledge of Content and Teaching (KCT) (7 items). The internal reliability for these items was above 55% for each knowledge domain (Herbst & Kosko, 2012). According to the creators' description, the items addressed content including: definitions, properties and constructions of plane figures including triangles, quadrilaterals and circles, parallelism and perpendicularity, transformations, area and perimeter, three dimensional figures, surface area and volume, and coordinate geometry (Herbst & Kosko, 2012). The online server provided reports for each teacher upon their completion of the MKT-G assessment. The test developers provided the difficulty level of each item using Item Response Theory parameters, with each of the 28 items receiving a score from 1 to 4. The overall MKT-G assessment was scored out of 99 total points. The scores for each sub-domain of MKT were as follows: common content knowledge – 27 points, specialized content knowledge – 47 points, knowledge of content and students – 13 points and knowledge of content and teaching – 12 points. With respect to these sub-domain scores, it is clear that MKT-G assessment had a high focus on assessing SCK as close to one-half of the possible points were attributed to items assessing SCK.

The second data source was a series of three semi-structured interviews. Semi-structured interviews included 1) conversations around a geometry task that participants were asked to solve utilizing GSP, and 2) questions designed to clarify and more deeply investigate events observed during my observations of class sessions. The geometry tasks asked participants to complete geometry tasks first without and then with the use of GSP. The tasks were designed to provide insight into participants' geometric thinking both with and without GSP, as well as participants' use and development of CCK and SCK. I administered these interviews during the second week of September, October and November. Each interview was audio and video recorded to capture the discussion between the participant and interviewer, as well as to document participants' actions as they engaged in the geometry tasks.

The third data source was a series of non-participant observations (Dewalt & Dewalt, 2002). With the instructor's permission, I observed each of the 13 class meetings of the semester. All observations except the first one were video recorded. In addition to video recording, I took observation notes according to an observation protocol, which I designed to capture major events and issues occurring for PSTs while they were interacting with GSP, with other materials or manipulatives, with the geometry content, and with the thinking and strategies of their colleagues. After the observation of each class meeting, I created write-ups and contact summaries for each class (Miles & Huberman, 1994).

Participants

As stated, PSTs participating in this study were graduate students seeking initial teaching certification for middle grades mathematics education. The PSTs had varied educational backgrounds, having previously earned bachelor degrees in areas such as: psychology, religion, business administration, economics, marketing, financial management, communication, electrical and computer engineering, nursing, physical sciences or engineering. All 16 PSTs enrolled in the course were invited to participate in the study at the start of the fall semester, and all volunteered to participate. The PSTs enrolled in the geometry course had little to no instructional technology experience or knowledge at the beginning of the semester. They also had not taken any geometry courses or reviewed geometry content since their experiences in high school. Only one of the 16 PST had any teaching experience prior to the beginning of the course. This teacher taught physical education, science and computer science for two years.

Out of 16 PSTs, 11 of them also took the methods course with the same instructor during the same semester. The methods and geometry courses were taught at the same day of the week. Although all 16 PSTs had agreed to participate in the study, I had decided to select six participants to participate in the semi-structured interviews. All 16 PSTs completed the entrance survey during the geometry course; 11 PSTs took the MKT-G assessment at the beginning of the semester during the methods course because the instructor did not want to spend more time for research during the geometry course.

I analyzed MKT-G assessment data in order to make sense of participants' initial MKT level as well as their beliefs about mathematics, teaching and technology. I used the results of these two assessments to create a knowledge and belief profile for each participant. Accounting for similarities in SCK and variation in responses to belief-related questions in the entrance survey, I selected six participants to interview. Table 4.1 demonstrates these six participants' *z*-score for the SCK domain of the pre MKT-G assessment and categorization of their beliefs about mathematics at the beginning of the study. PSTs' *z*-scores were calculated with respect to the average and standard deviation of 11 PSTs who took the MKT-G assessment.

To categorize their beliefs about mathematics, I used Ernest's (1989) framework, according to which participants were categorized as having instrumental belief if they signified mathematical rules and laws, but not their relationships. If participants underlined that mathematics required the relationships among concepts, I categorized their beliefs about mathematics as Platonist. Ernest's framework included a third belief about mathematics, problem solving belief, which is defined as the belief that approaches mathematics as a human construction. None of the participants indicated problem solving belief about mathematics in the entrance survey.

Participants Interviewed	**Pre-SCK Z-Score**	**Preliminary Views about Mathematics**
Cameron	-1.39	Instrumental View
Erica	0.68	Platonist View
Karl	**0.16**	**Platonist View**
Kathleen	**0.68**	**Instrumental View**
Kristin	**-1.13**	**Instrumental View**
Richard	**-1.52**	**Platonist View**

Table 4.1: SCK Z-Scores and Views of Six Participants Interviewed

Considering the richness of their data, proximity in their pre-SCK z-scores, and contrast between their beliefs, out of six participants, this paper documents findings for four focal participants: *Karl, Kathleen, Kristin* and *Richard*. I selected two PSTs having positive z-scores and two negative z-scores with different beliefs about mathematics (Table 4.1). Erica, Karl and Kathleen's z-scores for SCK at the MKT-G assessment were positive; out of these three, Kathleen was the only PST who had an instrumental belief about mathematics. Because of that, I selected Kathleen as one of the participants to proceed with for this paper. I selected Karl instead of Erica because his data were richer. Cameron, Kristin and Richard scored below average for the SCK domain at the MKT-G assessment. Out of these three, since Richard was the only PST who had a Platonist belief about mathematics, I also selected him as the third participant to proceed with for this paper. Finally I selected Kristin instead of Cameron as my fourth participant for this paper because of the richness of data from her throughout the semester.

Responses to open-ended questions in the entrance survey allowed me to create a preliminary profile for each PST about his/her background, expertise, professional experiences and general ideas about mathematics, teaching and instructional technologies. In the following paragraphs, I introduce the four focal participants of this paper:

Karl was 21 years of age and who completed his undergraduate major in economics. In the entrance survey, Karl viewed mathematics as a certain discipline independent of opinions, which requires critical and logical thinking. He also referred to this notion during the first interview by saying that "mathematics is a way of thinking logically and analytically … to apply it to [real world] situations." He also stated that mathematics cannot be attained simply by knowing all definitions, facts, procedures and related terms; understanding mathematics required knowledge beyond compartmentalized mathematical entities. In the entrance survey, Karl considered technology as a support for instruction, but not a replacement for it.

Kathleen was 35 years of age and came to the teaching profession from her career as a hair stylist. Before enrolling in the MAT program, she had professional experience in several areas such as management, barbering, cosmetics and interior design. She was a hairdresser for the last 12 years. When asked whether she recalled her geometry knowledge from high school years, she answered that mathematics, including geometry, was a part of her prior job: "There is a lot of angle involved in that when you're cutting hair. Actually the way that you hold it out from the head and how you cut the hair depends, determines how it's going to lay or move". At the beginning of the study, she stated in the entrance survey that, "mathematics is a subject consisting of foundational laws, which is objective; but the application of laws is subjective". In addition, she viewed mathematics as a more objective discipline compared to other disciplines. She explained how the meaning of a poem could be subjective as it can depend on who has read it. But, she considered mathematics as objective because of its accuracy and certainty. Even though Kathleen did not have a lot of experience with instructional technologies, her survey results suggested she viewed technology as something to be used for saving time during instruction.

Kristin was 53 years of age who worked as a nurse for many years before choosing teaching mathematics as a new career. Similar to the rest of the interviewees' experiences, she did not have any instructional technology experience in general or specific to dynamic geometry software. Regarding this, she stated she was not very comfortable with technologies, and preferred to do things "the old way" rather than with technology. At the beginning of the study, she viewed technology as a tool to make life easier. She gave a couple of examples from daily life to show she used technology in easing tasks, such as using electronic navigation systems instead of using a paper map or using cell phones for communication. However, these examples showed how she relied on the accurateness of the technology and believed in the results without any doubt. Similar to Kathleen's beliefs, in the entrance survey, Kristin thought mathematics required critical thinking. In her entrance survey, she referred to her children and how they were struggling with mathematics in school. Because of such experience with her children, she stated that she would make an attempt to change this point of view for struggling kids during her teaching. In addition to that, in the entrance survey, Kristin viewed mathematics as an objective discipline consisting of algorithms that can be applied in real life.

Richard was 43 years of age and who had previous work experience in business. He was the only participant interviewed that had teaching experience, not in mathematics, but in science and computers. Even though he had teaching experience in computer science, he also stated that his instructional technology knowledge is limited. When asked about his beliefs about mathematics, he identified mathematics as a certain subject with one correct answer, but available through a variety of methods. His beliefs about mathematics also indicated mathematics is a language that is common among different cultures. To elaborate on this opinion, Richard considered that mathematical premises, constructs and understanding would be the same everywhere over the world. In the entrance survey, Richard also stated technology should be integrated into instruction because of the needs of the society today, but with an emphasis on understanding while using it.

Methodology and Data Analysis

I used a multiple case study (Yin, 2008) as my methodological design for this study. Karl, Kathleen, Kristin and Richard served as the cases of analysis. Even though I investigated knowledge development and causal links between knowledge development and the factors affecting it, the study's main purpose was not to form statistical interferences, but to understand the phenomenon of knowledge development for PSTs while immersed within a particular technology. In this respect, a qualitative study, more specifically a case study approach, was the most appropriate methodology to examine factors and their relationships of real-life phenomenon, within a specific context.

Data analysis began with the MKT-G assessment. Having received participant's raw scores, I converted them into z-scores for each participant's pre- and post-assessment. The pre and post z-scores were used to represent the general MKT-G trend for Karl, Kathleen, Kristin and Richard during the duration of the course.

Write-ups from each class meeting were used to create an understanding of the pedagogy of the classroom. I used them to determine the frequency of technology use for the overall semester, who was using technology, the ways in which technology was used, and the frequency of teacher-led and student-led parts of the lessons. I also used the write-ups to identify important events addressing my research questions. In this case, I identified events in which I hypothesized there could be instances where participants developed aspects of SCK. I watched these video segments, and transcribed the discussion between PSTs and the instructor. I also examined course artifacts such as lesson materials, participants' class works and assignments to fill in the details of the events.

All interviews done with Karl, Kathleen, Kristin and Richard were also transcribed. Each participant's actions with GSP or on paper were narrated in detail by watching the interview videos. The responses to follow-up questions were linked to the transcription of videos and narrated events from class observations. In this manner, I began to compile data separately for Karl, Kathleen, Kristin and Richard from both the interviews and write-ups. The compiled data resulted in eight shared *Events*, each of which addressed a common geometry task that took place during interviews, class activities or whole class discussions.

All data for the eight Events for Karl, Kathleen, Kristin and Richard were analyzed using open coding (Corbin & Strauss, 1998) to identify: possible examples of SCK; if the identified example of SCK was newly developed by Karl, Kathleen, Kristin or Richard; and what factors, including the use of GSP, influenced their SCK. As soon as I had identified themes from the compiled data, I formed a rubric in order to assess Karl, Kathleen, Kristin and Richard's SCK development and the emergence of themes within the compiled data. I used this rubric in order to member check the data (Morse, Barrett, Mayan, Olson & Spiers, 2002) with Karl, Kathleen, Kristin and Richard, as well as with another PhD student in mathematics education to assess inter-rater reliability of the codes and interpretations. This process also provided an investigator triangulation through different evaluators (Yin, 2008). The inter-rater reliability of the coding for themes was 0.86. Finally, I compiled the *Events* during which themes were evident for Karl, Kathleen, Kristin and Richard, whether their SCK developed, and if GSP use was involved.

Results

In the eight identified Events common to Karl, Kathleen, Kristin and Richard, there were six events that involved the use of GSP and two events in which GSP was not used. For each event, I identified examples of SCK that could either be acquired prior or be constructed in the process of the event. Through the data analysis, I ascertained if evidence of this knowledge existed. Table 4.2 presents the number of events in which Karl, Kathleen, Kristin and Richard exhibited evidence of the identified SCK (or not), while using GSP (or not). Karl, Kathleen, Kristin and Richard displayed evidence of SCK during *four common events* where GSP was used as an instructional material. During two events where GSP was used, Karl and Kristin displayed evidence of the identified SCK, but Kathleen and Richard did not. During both of two common events through which GSP was not used, Karl was again able to display evidence of SCK, but Kathleen was not. Kristin and Richard showed evidence of SCK for different one of these two events during which GSP was not used.

		GSP Used (N=6)	**GSP not Used (N=2)**
Karl	SCK Evident	6	2
	SCK not Evident	0	0
Kathleen	SCK Evident	5	0
	SCK not Evident	1	2
Kristin	SCK Evident	6	1
	SCK not Evident	0	1
Richard	SCK Evident	5	1
	SCK not Evident	1	1

Table 4.2: The Number of Events during which Four Focal Participants Displayed Evidence of SCK with or without using GSP

Open coding process for eight events resulted in five major themes: 1) Opportunity to Justify Ideas, 2) Availability of Identified CCK, 3) Openness to Exploration, 4) Using the GSP for Exploration/Experimentation, 5) Viewing the GSP as a Learning Partner. In the following table (Table 4.3), I describe each of these themes:

Major Themes	Description
Opportunity to Justify Ideas	Independent of their correctness, the PST's ideas about geometrical principles, concepts, relationships, theorems, and/or procedures were challenged by the instructor, by the interviewer or by other PSTs so that they would justify their ideas. The event for each PST was coded with yes or no depending on the identification of any text where the PST's ideas were challenged and s/he justified them or not.
Availability of Identified CCK	The PST possessed the identified CCK related to the SCK targeted during the event. After CCK related to the SCK targeted for each event was identified, the event for each PST was coded with yes or no if the PST showed evidence of possessing it or not.
Openness to Exploration	The PST was open to exploration and experimentation while working on a geometry task. The event for each PST was coded with yes or no if the PST independently explored or experimented his/her idea on geometry or not.
Using GSP for Exploration or Experimentation	The PST independently used GSP to explore and experiment with his/her ideas. The event for each PST was coded with yes if s/he explored a personal conjecture. It was coded with no if s/he merely followed some pre-determined steps on GSP and did not explore a personal idea.
Viewing GSP as a Learning Partner	The PST viewed GSP as a tool to learn geometry with rather than as a tool for drawing or precise measurements. The event for each code was coded with yes if the PST stated that s/he viewed GSP for experimentation or conjecturing. The event was coded as no if the PST stated that s/he considered GSP as a tool for preciseness or drawing.

Table 4.3: Description of Six Major Themes

I next provide examples of these themes from within the eight common events of the four PSTs. I provide three examples: an event involving GSP in which four PSTs displayed evidence of SCK; an event that did not utilize GSP in which Karl and Richard displayed evidence of SCK and Kathleen and Kristin did not; and an event involving GSP in which Karl and Kristin displayed evidence of SCK and Kathleen and Richard did not. Each section exemplifies an event as representative of their themes. In Table 4.4, I describe these exemplary events and the date these events occurred during the semester. Because the example of the second event type also included participants who did not show evidence of SCK, I did not create another event type which might have been called No SCK Evident without GSP:

Event Types	Exemplary Events	Timeline
SCK Evidence with GSP	Square Construction with GSP	16-20 September 2013
SCK Evidence without GSP	Geo-board Activity	5 November 2013
No SCK Evident with GSP	Animation for an Inscribed Circle of a Triangle	18-22 November 2013

Table 4.4: Timeline for Exemplary Events

For the following three sections, I describe and exemplify each event type, and present findings for four PSTs with respect to the five themes. Afterwards, I share findings from the pre and post MKT-G assessment for CCK and SCK domains in order to compare my qualitative findings about these constructs with quantitative findings.

SCK Evidence with GSP

Out of six available events where the GSP was used, the four focal participants exhibited the identified SCK for four common events. Table 4.5 indicates three themes seem to be the most evident during SCK development with GSP for Karl, Kathleen, Kristin and Richard: 1) Opportunity to Justify Ideas, 2) Availability of Identified CCK, and 3) Using GSP for Exploration/Experimentation. I considered these themes as the most evident because their differences among four participants were lower than the other two themes. Regarding the description of these themes, each focal participant had to justify his/her ideas from challenges raised by other PSTs in the classroom, or by the interviewer during GSP tasks; had to demonstrate that s/he possessed the CCK required for the development of the identified SCK; and had to use GSP to explore a geometrical phenomenon or experiment his/her ideas with it.

	Karl	Kathleen	Kristin	Richard	*Variance*
Opportunity to Justify Ideas	4	3	3	2	0.67
Availability of Identified CCK	2	3	2	3	0.33
Openness to Exploration	4	2	3	2	0.92
Using the GSP for Exploration / Experimentation	3	2	4	3	0.67
Viewing GSP as a Learning Partner	2	0	2	1	0.92

Table 4.5: The Number of Events (of the 4 Common Events) Indicating the Emergence of Themes for SCK Development with the GSP

In the next section, I present the event, "Square Construction" as a representative example demonstrating how Karl, Kathleen, Kristin and Richard displayed evidence of SCK and of the themes attributed to them.

Square construction.

During the first interview, PSTs were asked to construct a square by using compass and protractor on paper. Then, they were asked to complete a task a middle school student started, and predict what the student was thinking with his/her procedure. According to the scenario, a student was working with GSP to construct a square. The student had started by constructing a circle and its radius and planned to continue the construction by using perpendicular and parallel line features of the GSP. Figure 4.2 demonstrates possible actions PSTs might follow in order to construct a square on GSP. After they completed the square construction task on GSP, the interviewer asked participants if they know any other way to construct a square on GSP.

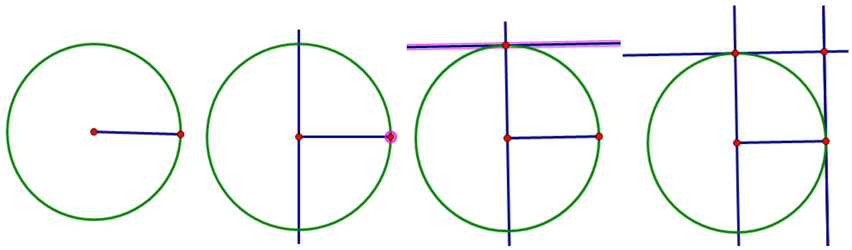

Figure 4.2: Expected Steps to Construct a Square on GSP

The identified SCK for this event was the prediction of the student's procedure to construct a square with the GSP according to the given scenario. Because the scenario did not specify which geometrical unit the student should use for the measurement of the edge length of the square, PSTs were expected to predict the procedure of this student's square construction in multiple correct ways. The scenario only restricted PSTs to focus on using the radius of the given circle, and affordances such as constructing parallel or perpendicular lines. With respect to the openness of the task, the square might be constructed inscribed in circle, circumscribed around the circle, or from the radius given as its side length. The required CCK for the SCK was the knowledge of square having four congruent sides and each internal angle to be ninety degrees.

Kathleen. Kathleen used compass to have the same length for each side of the square on paper. Her drawing started with a line segment she drew on the paper with straight edge of the protractor. After that, she set the compass to any convenient width and scribed an arc above each side of the line segment she drew. She scribed the arcs in such a way that the arcs were approximately perpendicular to the line segment. However, she did not use protractor to measure the angles. She seemed to possess the required CCK, which would allow her to solve the given tasks in the right way. For example, by setting the compass with the same width for both sides of the line segment, I concluded that Kathleen's approach suggested she understood that squares would have four equal sides and four right angles.

Next, Kathleen used GSP to construct the square, beginning by creating the picture in the student's work in the presented task. By using the drawing tool for segments, she extended the radius of the circle to make it a diameter, but the diameter she constructed was not a straight line (see the first figure at Figure 4.3). Then, she constructed a perpendicular line to the diameter through the center of the circle. After identifying the intersection points of the perpendicular line with the circle, she connected the four points that lay on the circle with line segments, which resulted in her "square" inscribed in the original circle (Figure 4.3).

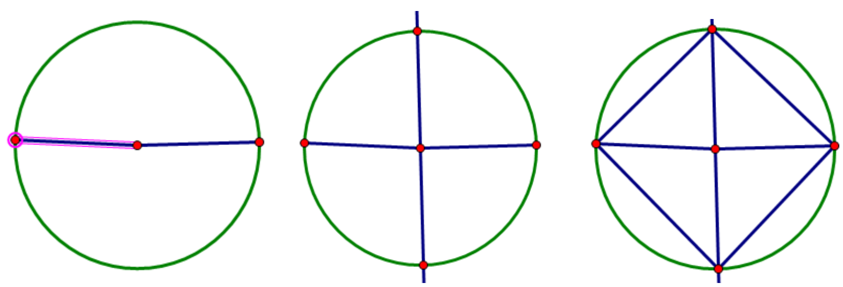

Figure 4.3: Kathleen's Square Construction

Kathleen's "square" was not constructed correctly because she did not originally construct a straight line through the center containing the radius. As a result, she could not always keep the constructed object as square when she moved the points on its vertices. At this point, she did not use an affordance of the GSP to have a diameter precisely, but used eyeballing for the internal angles of the square the same as she did on paper. For example, she might have constructed a parallel line on the center to the radius given so that the diameter would have been a precise line segment rather than an approximation. During the interview, the interviewer challenged her ideas for justification about her steps to construct a square on the GSP, which allowed her to find her error and correct it.

Interviewer:	Okay. So you say that now is this a square. That one is the square. How do you know that we can keep it as square if I play with points here?
Kathleen:	I think it would be a square. What do you mean?
Interviewer:	[The interviewer dragged the radius she constructed.] See, it is not a square anymore… Why might be the reason?
Kathleen:	Oh. Let me think. It might be off just a little bit because I didn't come, I didn't like extend that line perfectly. But yeah, it's close… If it was the extension, then...there's no problem with that one I believe. So… If I… How can it be a straight line perfectly?
Interviewer:	How would you do that?
Kathleen:	If I construct a straight line here [the center]… I already have the radius… [She constructed a straight line overlapped with the radius given]. Now it is perfectly straight.
Interviewer:	Now it can be a perfect square.

Kathleen's work with paper before GSP, where she emphasized perpendicularity and congruency of the edges, showed that she had already had this CCK. She knew that a square had to have four equal sides and four ninety degree internal angles. Her exploration with the GSP along with my guidance to challenge her ideas enabled her to develop the identified SCK during the interview. During the first interview however, Kathleen stated her intent to have students first create constructions on paper, and then to use GSP for "polish":

Interviewer:	What do you think about constructions in general like as a learning goal for geometry?
Kathleen:	I would still guide students to use the archaic method to start and then...probably go into something like GSP to polish it and finish it, to perfect it. But to...like to explain parallel and perpendicular and stuff like that, I'd rather let them conceptualize in their minds. Because, to me, this [GSP] gives too much.

This belief about only using GSP as a tool for precision seemed to limit her ability to see GSP as a learning partner. Even though Kathleen was able to develop the SCK for this task as a result of having her ideas challenged by the interviewer, and this pushed her to explore geometry with GSP, her statements indicated that she did not view GSP as a tool for exploration, but as a tool used to create precise measurements. At this stage of the course, Kathleen's explorative experiences with GSP did not change her view about GSP.

Karl. On paper, Karl constructed a rectangle by using the procedure to construct a perpendicular bisector. He constructed a perpendicular bisector on a line segment he drew, and kept the same procedure for each line that was perpendicular to the previous one. However, he did not manage to construct a square with compass and protractor, but a rectangle (Figure 4.4).

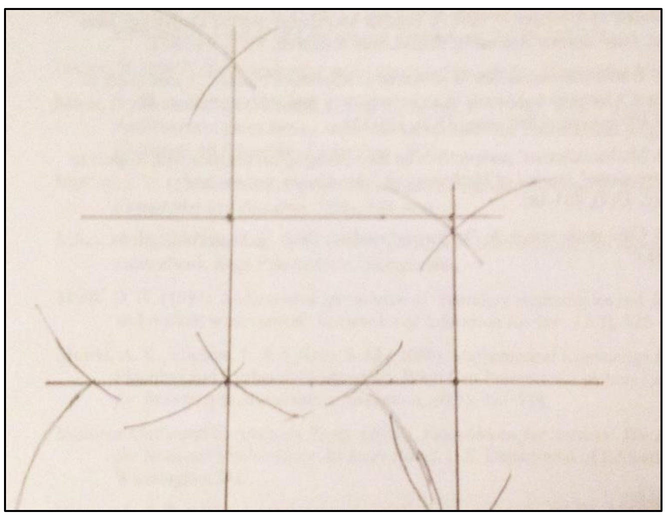

Figure 4.4: Karl's Rectangle Construction on Paper

He mentioned to draw a tangent line, an inscribed circle in a square, but did not know how to do them and work on his ideas. The interviewer challenged these ideas as a transition to work on them with GSP.

Karl: Well I knew, I was thinking, well I didn't think square. I was thinking rectangle. And so I know that we know how to draw perpendicular bisectors. You know perpendicular bisectors will have a right angle.

So if you just do one on that one. You can keep going all the way around.

Interviewer: But for the square?

Karl: Yeah, I forgot how to make sure all the sides are equal. Okay, it's a circle. And what I was going to do now was to draw tangent lines. That wouldn't work either. Never mind. I was going to make an inscribed circle. Like I was going to do like an inscribed circle and then have a square around it.

On GSP, he tried out his idea of inscribed circle of a square. He first constructed the diameter from the radius by drawing a line on the radius. Different from Kathleen, he drew a parallel line to the radius on the center, which resulted in a precise diameter rather than an approximation. Then, he constructed tangent lines to the circle by using the perpendicular line construction feature of the GSP. Next, he constructed parallel line to the perpendicular lines, and finalized the construction of a square circumscribed around the circle (Figure 4.5).

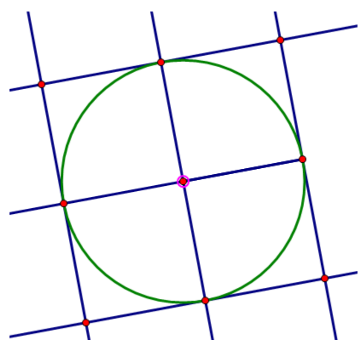

Figure 4.5: Karl's Square Construction on GSP

Karl's CCK for the definition of square was available during his square construction with GSP the same as Kathleen. His practice with mathematics indicates his flexibility to try new ideas to see if they work or not. This flexibility with mathematics also allowed him to use GSP as a learning partner rather than just a tool for precision. GSP seemed to help Karl's SCK development more than on paper. He predicted the mathematics behind the student's square construction procedure by demonstrating his square construction with the GSP. Different from Kathleen, he considered that GSP was conducive in learning geometrical concepts more than paper:

> Personally, I prefer the software only because I hate compasses. And I just, they wobble and everything. After you build it, I like being able to play around with it and seeing how it changes. I guess a lot of people from what I heard, they like, they think you get more conceptual stuff by doing it on paper. But I think you can get just as much on the GSP. (Karl, first interview)

With his statement, Karl viewed GSP more as a learning partner rather than as a tool to demonstrate paper-constructed models. His openness to exploration with geometry seemed to be enhanced with the software because of its affordance to dynamically represent how personal assumptions about geometry might be generalized or refuted. With the square construction activity, he evaluated whether it was feasible to construct a square out of four tangent lines around a circle. His procedural reasoning began to emerge on paper, but he recognized that he could not assess the validity of his reasoning on paper. This limitation guided him to work on his reasoning within a dynamic geometry environment. His preference to use the GSP and views about the GSP also benefitted his conceptual gains, which resulted in his SCK development. He could not manage to construct a square with paper and pencil even though he had ideas how to do it. Before he was asked to work on GSP, he mentioned his idea of a square construction circumscribed around the circle, but he did not work on this idea on paper. GSP allowed him to explore his ideas to construct a square in a better way than on paper, and to see if his prediction about the mathematics behind the student procedure was correct or not.

Kristin. Kristin's procedure to construct a square was different from Karl and Kathleen's procedures (Figure 4.6). She first constructed a point on the circle so that she could create a second radius that would be perpendicular to the radius given. Instead of using the construction of a perpendicular line affordance of GSP, she eyeballed the location of this point in a manner similar to Kathleen's approach. She recalled one of her colleagues in class had mentioned the construction of perpendicular line command and wanted to construct a perpendicular line to the radius given and that was passing through the center of the circle. She selected the random point she had constructed on the circle and the original radius for this procedure. As a result of "eyeballing" the place of the point she first constructed, she had to move it onto the perpendicular line so as to make it visually perpendicular to the radius given. Next, she constructed two other perpendicular lines to create a "square" constructed from the radius given. When she finished her construction, the polygon approximated the appearance of a square.

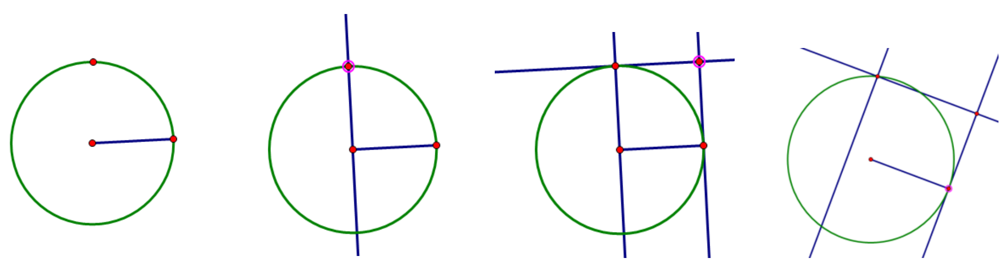

Figure 4.6: Kristin's Square Construction on GSP

Kristin was aware of the mathematical properties required for a polygon to be a square. After she finished her construction, she wanted to assess its validity by referring to the congruency of its sides and each internal angle to be ninety degrees. I considered her reasoning after her construction as evidence of CCK required for SCK expected during this event:

> I know that, if tested, these angles… I could measure the angles… of each angle, and make sure that it is ninety degree. But, I went from what I have and created with perpendiculars… So I am pretty confident that that is a square although I can verify by measuring each side if you want me to do that. (Kristin, first interview)

When the interviewer moved the randomly constructed point, the construction fell apart and demonstrated her construction would not hold its features in every case. Rather than spotting the point on the circle from the perpendicular line she constructed, she randomly constructed it. Because she did not know that there was an error with her construction, the interviewer used her error as an opportunity to ask her to justify her ideas and asked what the reason could be behind her construction error:

Interviewer:	If it is a square construction, whenever I play with the points, it always has to be a square. But see… do you see what the problem is (*see the last figure at Figure 4.6*)
Kristin:	Well the problem is, it is not staying in the square when you are manipulating it. And ideally it should. Now why is it not?
Interviewer:	It must be because this point [the first point she constructed] is not exactly perpendicular to the center. That seems to be the one that is not maintaining its position…

The interviewer kept questioning Kristin's reasoning behind her construction so that she might correct it. After a while, Kristin recognized that, instead of eyeballing, she could utilize the intersection point of the first perpendicular line she construction with the circle. Even though Kristin corrected her construction, she did not try to explore other ways to construct a square with GSP. She only constructed the square according to the scenario given during the interview, but not beyond it. In a way, instead of trying to find multiple ways, Kristin was more focused on finding an answer for the problem given. Because of this approach during the first interview, I coded that she was not open to explore mathematics. In addition, her views about technology during the first interview underlined its potential to demonstrate concepts in different ways with more visual appeal. However, she did not clearly view GSP as a learning partner:

> I think it is a tool if used wisely. For example, our school lets students to go to computer lab including a software that coordinates lessons. So, I don't know the nature of that and how it is going to look like. But it is different than the teacher stands up there and talk from a smart board. And […] sketchpad may enable you to create something in a different way. Even PPT, or reporting things together, it is much graphically appealing now… visually appealing. (Kristin, first interview)

Regarding this statement, I would say that Kristin was skeptical about the software's potential to impact student learning. It seems that she had an interest to integrate technology for the sake of students' learning, but she had not learned enough about how to use it at this phase. She favored its potential to visualize things in a better way to engage students.

Richard. Richard's square construction (*see* Figure 4.7) was similar to Kristin's construction. He started his construction with a second radius visually perpendicular, but not constructed to be perpendicular, to the radius given. Next, he also constructed two other perpendicular lines to create a "square" constructed from the radius given.

However, the same as Kristin, his construction could not hold its features when one of its vertices was moved (*see* the last figure at Figure 4.7):

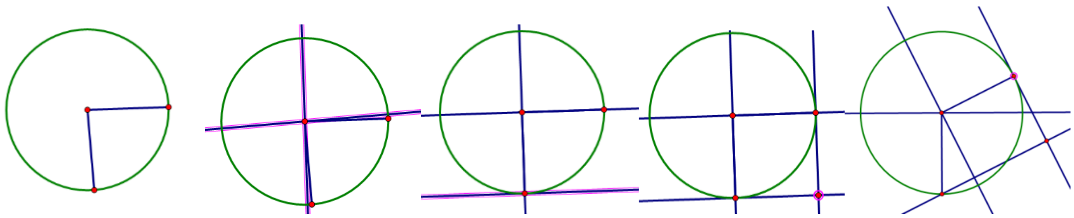

Figure 4.7: Richard's Square Construction on GSP

In order to uncover his reasoning, the interviewer asked him what the reason might be for this error. Richard thought he made an error because he did not highlight the right line or line segment in constructing his perpendicular lines. He did not recognize that the reason for his error was his approximation of the radius that was supposed to be perpendicular to the first one:

> Interviewer: So you have a point there as well. So let me try to see if it is a square or not. [Pause]. It is *almost* a square. I see your point. What might be the reason for this error?
>
> Richard: I think I highlighted the wrong line here. That should be the radius. Let's see. Let's take this back. So that works. And then I can't say which line we have to remove.

Even though his reasoning was different from what I expected, he still corrected his construction by removing lines he constructed and starting a new construction from the beginning. During his second trial, he managed to construct a square that maintained its features when a corner was moved. His error during his first trial allowed him to justify his ideas about how to remove the error. Although the GSP allowed him to explore his ideas more, he did not explore any different techniques beyond what the scenario asked.

Richard's view about technology was similar to Kristin's views. He was skeptical about its advantages compared to its disadvantages. During his interview, he emphasized that technology should not be used for the sake of entertainment, but for students' learning. He also showed concern about the use of technology as a permanent resource in the classroom:

> It is a wonderful tool like everything else. It has its benefits, and disadvantages. And we can get into the entertainment business rather than teaching business. We have to make sure that students are learning, not just being entertained… Technology has the ability to make a world different but it also has the ability to inhibit them learning as we talked about earlier. If the kids learn a computer and don't know in their minds that two plus two is four, but the calculator says it. (Richard, first interview)

Richard seemed open to the concept of technology as a learner partner. I considered the statement above as he viewed technology as a learner partner because he stated that he emphasized learning with technology rather than entertaining with it. In other words, it is not a tool for him to fancy kids. He might view GSP as a learner partner as his ideal; but to practice GSP with such a view might require his learning experience with it more because he is also suspicious about its effectiveness to learn basic mathematics such as arithmetic.

SCK Evidence without GSP

Karl, Kathleen, Kristin and Richard were not required to use GSP in two of the identified events. These class activities were not designed to use GSP, but rather other materials such as geo-boards or the class textbook. Within these two events, even though the four participants were engaged in justifying their mathematical ideas through interactions with their colleagues or the instructor of the course, it was not sufficient for Kathleen, Kristin or Richard to develop the expected SCK. Kathleen did not develop her SCK during either event where the GSP was not used. Karl displayed evidence of SCK in both events. Kristin and Richard each developed SCK during one of the two events, but not in the same one. When I looked at the themes emerging from their data for two events (Table 4.6), I recognized that one major difference addressed the availability of CCK. While Karl and Richard displayed evidence of the required CCK for the identified SCK during these two events, Kathleen did not have CCK, which might have hindered her SCK development. Kristin did not have CCK required during one of these two events. Finally, unlike Karl, I did not observe evidence of openness to exploration with Kathleen, Kristin or Richard during these two events. While Karl mathematically reasoned through the given tasks through "what if" questions, Kathleen, Kristin and Richard did not utilize these exploratory questions. To exemplify the difference among Karl, Kathleen, Kristin and Richard in terms of their SCK development, I describe one of these events called Finding Area of Triangles using a Geo-board and focus on how two themes, Availability of Identified CCK and Openness to Exploration, emerged differently for the four participants. Variance for these themes among four participants was larger than the other themes.

	Karl	Kathleen	Kristin	Richard	*Variance*
Opportunity to Justify Ideas	2	2	2	2	0.00
Availability of Identified CCK	2	0	1	2	0.92
Openness to Exploration	2	0	1	1	0.67
Using the GSP for Exploration / Experimentation	NA	NA	NA	NA	NA
Viewing GSP as a Learning Partner	NA	NA	NA	NA	NA

Table 4.6: The Number of Events Indicating the Emergence of Patterns Themes for SCK Development without the GSP

Finding area of triangles using a geo-board.

During the geo-board class activity, the participants, along with their classmates, were placed into groups of four and asked to find all possible triangles having an area of 2 square units on a geo-board. Kathleen, Kristin and Richard were in the same group; and Karl was in another group. The instructor distributed geo-boards and strings to each group member. After they had individually sketched their triangles on their geo-boards, they compared, contrasted with one to another, and evaluated whether the area of the triangles was exactly 2 square units or not. After they discussed their individual works, each group presented a group work on the board to the whole class.

The intended SCK associated with this task was understanding the mathematics behind a group of PSTs' geo-board work. One group showed all possible triangles having 2 square units on their geo-board to the whole class (Figure 4.8). This group was trying to use all possible base and height combinations on the geo-board. They were also trying to keep the base and height the same by moving one vertex of the triangle. In other words, they were trying to show that translating the top vertex of a triangle along a parallel line would not change its area. During the third interview, the four participants were asked to make sense of the mathematics behind this group's procedure. They were expected to understand what their classmates were doing within their group work. To develop this SCK, I again considered participants needed to understand the existence of an external height to an obtuse triangle, and the formula to find the area of a triangle.

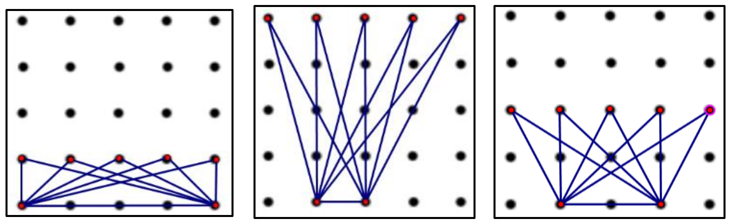

Figure 4.8: All Possible Triangles Having Two Square Units

Kathleen, Kristin and Richard. In their group, Kathleen, Kristin and Richard sketched a collection of triangles on their geo-board, some of which had an area of 2 square units and some that did not. They created a triangle having a base of length 2 and height of length 2, a right triangle having four units of height and one unit of base, a triangle having three units of base and one unit of height, and a right triangle having three units of base and one unit of height (Figure 4.9). After examining their triangles and discussing them, the group decided the triangle having three units of base a one unit of height would have an area less than 2 square units.

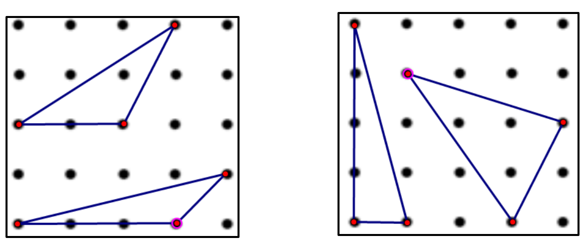

Figure 4.9: Kathleen, Kristin and Richard's Group Work with Geo-board

As evident from the example below, Kathleen was missing the requisite CCK, which resulted in her failure to develop the identified SCK. During the third interview, Kathleen explained she used estimation and approximation as a method to find the area of triangles with two square units:

Interviewer:	During the activity that you used the geo-board, how did you start to solve the problem?
Kathleen:	I had a visual of the unit in my mind. I could see ways of breaking up the unit in half. I saw a way of breaking it into quarters. And so, I estimated. I'm all about eyeballing space and estimating how much that is, and then either turning that into a rectangle or another shape. So when I stretched the rubber band, I just tried to keep in mind that amount of space that I had saw. And saying okay well I have two-halves there, either because that was directly on the diagonal or because the way that it split...a two-unit plan.

Her estimation of half unit of a square was understandable, but her use of a quarter unit of a square did not make sense. Apparently, she approximated spaces less than half unit as quarter unit. In order to understand her colleagues' procedures, Kathleen needed to understand there are three ways to create a triangle whose area is 2 square units on geo-board: 1 unit base, 4 unit height; 4 unit base, 1 unit height; 2 unit base, 2 unit height. However, Kathleen stated she was not able to identify the three basic triangles with an area of 2 square units. Further, when asked to consider the work of the other group, who presented all possibilities, Kathleen was also not able to explain why their process worked.

Interviewer:	Did you come up with any possibilities by using pegs and strings for the area or just a couple of them, not all of them?
Kathleen:	I didn't get the three basic shapes that they ended up doing. I guess they translated the top points of the triangles. Eventually they ended up with like using the same base and then they just moved the point five different times, like stretched it across a parallel line.
Interviewer:	Why were they doing that?
Kathleen:	I suppose because it created different leg lengths. The base stayed the same the whole time.
Interviewer:	Base staying the same. What do you mean by leg lengths?
Kathleen:	Like coming down from the top point to the corners of the base. The base stayed the same and the area would stay the same
Interviewer:	Because of?
Kathleen:	Kathleen's postulate. I'm pretty sure they stay the same, but I don't know how I know. I think they all ended up being the same.

Kathleen seemed to have a lack of sound conception about the height of a triangle, especially when it was external of an obtuse triangle. This lack of conception also limited her to understand that she can create multiple triangles having the same base and area while translating the top point on a parallel line. In addition, she did not explore different triangles as much as other PSTs in the classroom.

Kristin's third interview showed that she also did not understand what her colleagues were doing with their sketch on the board. Her statement showed that she believed that they were keeping the area the same, but without any rationale. It was an instinct rather than a thorough understanding:

Kristin:	I don't remember anything other than the fact that they were changing one peg at a time. So they changed it one way. It changed the area going the other way by the same amount.
Interviewer:	How? Did you consider that the area was still 2 units while they were changing it?
Kristin:	I believe they are
Interviewer:	How?
Kristin:	Because they are not changing… As they are changing, they are changing one and they are changing the other by the same amount.
Interviewer:	Changing amount of what?
Kathleen:	The amount of area. I am trying to remember. I am remembering the drawing but I have trouble remembering the context of all of that.

Kristin's interview did not show evidence that she had the required CCK. She did not relate the same area to the same amount of base and height, but only referred to visual changes on lengths of its sides with the movement of its top point. As a result, she did not understand the mathematics behind her colleagues' procedure and drawing on the board and did not demonstrate the SCK expected. Kristin's exploration of geometrical principles was also limited during this event. She did not explore how to find triangles having 2 square units in different ways as much as Karl or Richard.

The third interview with Richard indicated that he explored how to form triangles having 2 square units on geo-board. Even though they were in the same group, he seemed to have a better understanding and reasoning about how to find the area of a triangle from other shapes on his geo-board. His reasoning to find the area of a triangle from square was correct but limited. He preferred to make triangles having 2 square units from squares having 4 square units:

> My thought was making everything square. So if you may have the triangle, you make a square. And then, then you knew the area... And you knew the triangle would be half of that square. So, for example, if you made a triangle, and I made the square and it covered four squares completely, then we know the area is half of that. So that the area is two. So then we could elongate it, make it four long and run a triangle... Basically I was boxing things, make it a square and take it half of that area. So then, we were sure. (Richard, third interview)

Richard's reasoning was understandable, but not sufficient in order to find all possible triangles having 2 square units by himself. He did not think to generate triangles from parallelograms as well. In addition, separating the square into two would only create right triangles. Even though he did not come up with all possible triangles, his interview showed that he had the required CCK and demonstrated SCK expected for this event:

> The half of the base times height... So we had moved the radius two units up, so any string along there, as long as this base stays the same and that height stays the same, it did not matter which way you moved the angle. (Richard, third interview)

Richard understood that his colleagues were trying to keep the base and height the same by moving the top point of the triangle along a parallel line. I considered this statement as evidence for his conception of external height of an obtuse triangle, which indicated that he had the required CCK. As a result of his CCK, he managed to demonstrate the expected for this event during class meeting and the third interview.

Karl. Karl's individual geo-board included three main triangles: one having a four units of base and one unit of height, one right triangle having two units of base and height, and one right triangle having four units of height and one unit of base (Figure 4.10). He considered that there should be three types of triangles, but the triangles displayed on his geo-board did not include all possible triangles.

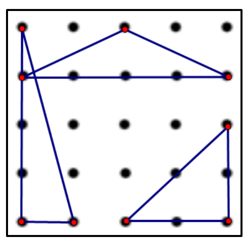

Figure 4.10: Karl's Group Work with Geo-board

Karl explained during the third interview that he had come up with the same idea his classmates had demonstrated on the board. He had used the area formula while doing it in his small group work.

Interviewer:	So a couple of your colleagues has done this. On the board, they demonstrated their method, which included several triangles having the same base and having different angles and direction. So what do you think they were doing with that one? What was the math behind it?
Karl:	I think they would work because if it's one-half based times height, so we, as long as we kept the same base and the altitudes the same, your areas are all going to be the same no matter what the angle measurements are or anything like that. So it does some, it sort of just showing that there's a wide variety of triangles you can draw that have the same base and the same altitude.

Karl's interview also showed evidence that he was open to exploration even while working with the geo-board. When he was asked to explain what his classmates were doing with their methods on their geo-board, he stated that he came up almost with the same idea and shared his enthusiasm with his group-mates:

> Interviewer: When they showed this thing on the board the first time, do you just understand it at that point or you didn't know before it...what they were doing?
>
> Karl: I knew before what they were doing. Because I kind of had come up with the same thing in my group. I sort of did like a one base and a four height one. And I was saying, oh look if I keep moving it over I still have the same height, or same altitude, no matter where I move it. So there's one, two, three, four, five possibilities. Let's move on to the next one. So I kind of came up with that. Not maybe using the same words as them, but I had the same idea that as long as you keep the altitude and the base the same, your area stays the same. I was excited. I was like, "look what I found out".

Karl understood the CCK associated with this task, as he was able to identify the three "core" triangles using the area formula for a triangle. Further, Karl described how he moved the string attached to the top point of the triangle left and right without changing the height, which showed that he considered the height of a triangle might also lie outside of a triangle. I coded these statements as evidence of the availability of his CCK. As Karl went on to search for *all* of the possible triangles with area of 2 square units, he had also generated the expected SCK. I cite his stated excitement concerning his work as evidence of his discovery. Karl's attitude towards mathematics and his openness to exploration in this task was similar to his work with GSP. As in the other cases demonstrated, this attitude was in contrast to Kathleen's.

No SCK Evident with GSP

I next present the "Inscribed Circle of a Triangle" event, as an example in which Kathleen did not develop the identified SCK, but Karl, Kristin and Richard did. Each one of them demonstrated SCK in a different way. Table 4.7 presents the themes attributed to Karl, Kathleen, Kristin and Richard in this event. While four participants' mathematical ideas were challenged during the interview for justification, and the GSP gave them an environment to explore a geometrical relationship, these two conditions were not enough for each participant to develop their SCK with the GSP.

	Karl	Kathleen	Kristin	Richard
Opportunity to justify ideas	Y	Y	Y	Y
Availability of Identified CCK	Y	N	Y	Y
Openness to Exploration	Y	N	Y	N
Using the GSP for Exploration / Experimentation	Y	N	Y	N
Viewing GSP as a Learning Partner	Y	N	Y	Y

Table 4.7: The Occurrence of Patterns Themes during the Inscribed Circle of a Triangle Event[2]

Area, perimeter and the inscribed circle of a triangle.

During this event, each of the four participants was asked to find the relationship between the area of a triangle, the perimeter of the triangle, and the radius a circle inscribed in the triangle first on paper, then by watching animations built using GSP (Figure 4.11). During the animation, the original triangle folds out into three triangles where their height is the radius of the circle. Next, the top vertex of each triangle moves along a parallel line towards to the center of the circle.

[2] Y represents the occurrence of the theme by the participant, and N represents that the theme was not emerged for the participant.

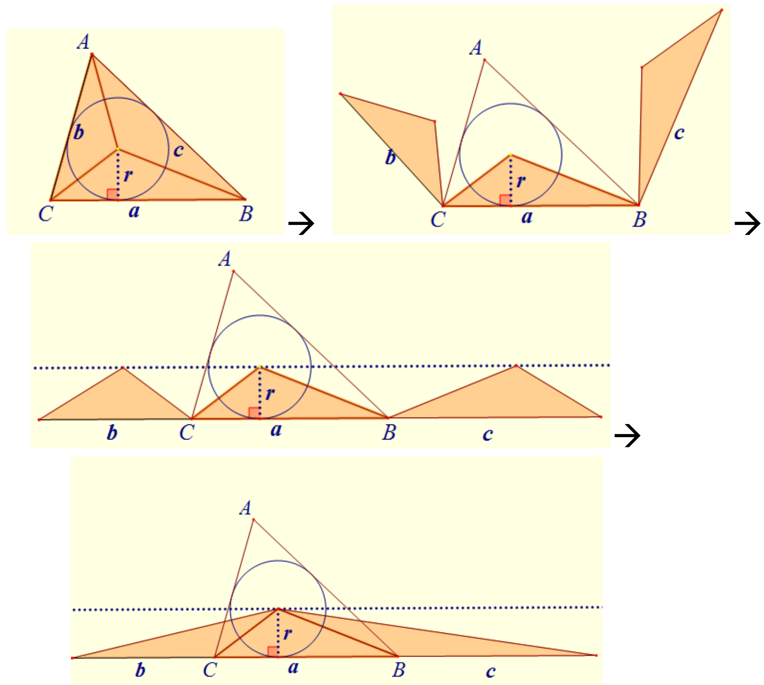

Figure 4.11: Screenshots from the Animation for the Inscribed Circle of a Triangle

The identified SCK for this event was the mathematical knowledge required to understand the procedure conducted with the animations. The animations demonstrated that the triangle can be separated into three triangles, each one having a base length of one of the sides of the original triangle and a height equal to the length of the radius, as the radii are tangent to the sides in an inscribed triangle. Further, a new triangle can be formed from these three triangles where the base would be the perimeter of the original one, and the radius of the inscribed circle would be the height. To understand it with the animations, I considered participants should understand the height of an obtuse triangle in some cases, lies outside of the triangle (Figure 4.12). This knowledge was identified as CCK required for the development of SCK.

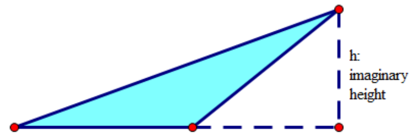

Figure 4.12: Imaginary Height of an Obtuse Triangle

Karl. Karl presented evidence of this CCK by identifying the external height of a triangle. As a result of his understanding of the external height of an obtuse triangle, Karl was able to make sense of the animation and construct the relationship between perimeter and area and the inscribed circle of a triangle.

Interviewer:	The top points of these three triangles are moving on a parallel line, what's happening here? What's the logic behind this movement? That way, do we keep the original small triangles the same?
Karl:	The areas aren't changing. It's just the shape of the triangle is changing.
Interviewer:	Why do you think that the areas are not changing?
Karl:	We didn't change the area because it didn't gain or lose any. We kind of just skewed the triangle back over.
Interviewer:	What are the parameters that we keep the same then?
Karl:	The base.
Interviewer:	What else do we keep the same?
Karl:	The altitude. So the base and the altitude of the triangles stay the same. So one-half base times height. So the area will still stay the same. The radius is the altitude of the three smaller triangles. So the radius could be the height. Oh, okay, I see it now. So if you add, cause of the new base, okay so if you want to find the area of this triangle. It would be b + a + c times one-half of the radius. So you could do b + a + c times one-half times r, the radius and your perimeter would also just be b + a + c. Area of the triangle would be equal to the perimeter times the radius divided by half.

The dialogue between the interviewer and Karl showed evidence that he understood the mathematics behind the animation. He thought that the reason behind the movement of the top points on a parallel line was maintaining the areas the same. Step-by-step, he also constructed the relationship between the area, perimeter of the triangle and the radius of the inscribed circle. The final part of the interview also showed how he found GSP for his learning:

> If you would have just handed that, I would have just stared at it all day. But seeing animation definitely helps because you can see...once you get, even thinking about rotating it out, I would have never thought about that… Yeah, like the rotating out. I would have never have thought to put the three triangles as one big triangle. I would have never thought to do that. So to see that helped me think that okay the perimeter is your new base, so that was really helpful. (Karl, third interview)

Karl's statement above indicated his view of GSP as a learning partner. He was aware of how the program was advantageous visually and how it might have been hard for him to do these multiple tasks by himself on paper. Kathleen's experience with GSP during the same event was quite different from Karl's experience.

Kathleen. Kathleen however, could not make sense of the animation, as she was seemingly "stuck" with the idea of the height of a triangle lying outside its interior. When the interviewer recognized that Kathleen seemed to be missing the required CCK, he used GSP to demonstrate the area would not change when the top points of the triangles were moving on a line parallel to the base. However, Kathleen did not understand what the logic was behind the animation in terms of geometrical principles. Kathleen's lack of understanding of an external height hindered her SCK development for this task:

Interviewer:	…on the parallel line. Can you see why they've done this?
Kathleen:	No. I don't know why they did that. It made sense when they were in points over here [before moving along the parallel line]. But once they elongated it, I don't have no idea why.
Interview:	Okay. So how would you find the area of that triangle then? What is the area formula for that thing?
Kathleen:	It's the same thing, but not the, I don't know what this height is… It still seems to me like they, when they move that point and they stretch the shape, the area would change.

Interviewer: Do you think that? So you're not convinced with that kind of animation?
Kathleen: No. I am not sure why I think that.

After that, the interviewer informed Kathleen about the concept of external height, and showed how the area would be the same when the top point of a triangle moves on a parallel line to its base. Even though she grasped the point, she was not convinced about the animation and the geometric principles behind it. Kathleen suggested she would have better understood the idea without GSP.

Interviewer: Is this animation conducive to understand those things?
Kathleen: In a slow painful way I did. I might have seen that faster on a geo-board. I was just stretching that point over, I would have recognized the same area because I would have had the grid. It would have clicked a little bit faster. Somehow [on the GSP], it changed when the point was here and it slid over here, and as it elongated, it seemed like it went down. It didn't seem like it was parallel. It seemed like the height changed as it slid over. I would have kept it [the triangle] together. For some reason the breaking it apart just like blew my little brain.

Kathleen's statement indicated she did not consider GSP as a learning partner, but as a tool that actually impeded her learning. Kathleen seemed to attribute the reason for her lack of development of the SCK in the task to GSP. During the interview, the interviewer explained the top points of the three triangles were moving on a line parallel to the base of the new triangle. However, Kathleen did not think the line was parallel and that the height of the triangles was changing. Karl was introduced to the animations with instructions identical to those given Kathleen. While he was able to connect the parallel line to the area of the new triangle, Kathleen could not see it using GSP. Her SCK was not developed with GSP. Both its use as a representation as well as her beliefs about what it was useful for might have been the reason for this result.

Inscribed Circle of a Triangle Event showed that Karl could develop SCK for the task because he had the requisite CCK, and viewed GSP as a learning/exploration tool. On the other hand, Kathleen did not meet either of these requirements, and thus could not construct the SCK during this event.

Kristin. As mentioned, Kristin did not demonstrate the required CCK for the geo-board event, which was the knowledge about external height of an obtuse triangle and its area. This event also required the same CCK because the animation used the same geometrical premise at some point. Kristin's interaction with GSP showed that she was still lacking that knowledge. By instinct, she knew that the area of internal triangles after their rotation had to be the same, but she did not know how. Her mathematical reasoning on the animation allowed her to develop that CCK during the interview:

Kristin:	I would say that the height is the perpendicular line at... or somewhere where it has a highest point with a perpendicular line. The height is not the highest point of the triangle, it is the height of the perpendicular line, I believe. I wonder… I know it [r] is the height of the center one because it is a perpendicular line… that is where I am getting hung up. I am used to height as a perpendicular line. If I go from here to here, it would be the height… Yeah. So that would be the height, They [the areas of internal triangles] would be the same. Because you still have this base.
Interviewer:	So, can you see the relationship right now, between area, perimeter and r?
Kristin:	Oh, b plus a plus c is the perimeter… Well the area of the triangle that you opened up is ½ times the perimeter times the radius.

As a result of her reasoning and development of CCK required for the task, Kristin managed to understand the mathematics behind the animation, construct the relationship among the area and perimeter of the triangle, and the radius of the circle inscribed in it. Through interactions and support from the instructor, she was able to demonstrate the SCK expected for this event.

At the end of the interview, the interviewer asked whether the GSP animation was conducive for her understanding or not. Kristin was not skeptical about GSP in the same as Kathleen. She found it useful because it allowed her to re-watch the animation:

> I think it helped being able to do it and undo it so that I could repeat. If I do it on paper, then I have to draw the whole thing again. But if I can see it multiple times, then I can watch the angles move and I can say even though the angles changed, the area did not change... One thing I liked about GSP is that it is very… They don't really skip steps. They really show you step by step by step... I like that because I am not sure if somebody tells to construct this relationship, then I could have done it as well as you could. (Kristin, third interview)

Kristin used GSP as a learning tool. Her experience during this event enabled her to learn content that she had not before. However, because her experience with this technology was limited, she did not consider its uniqueness coming from dynamic visualization and potential to reason in multiple ways. She only viewed GSP as a tool which allowed her to measure, draw and animate. These affordances might also be labeled as its advantages for learning, but not with its full capacity.

Richard. As soon as viewing the animation, Richard recognized that the area for each triangle would be the same. He understood it by relating the height of each internal triangle to the radius of the circle inscribed in the triangle, but he did not relate it to the parallel line demonstrated:

> Well I see one is using the radius, and that would make sense because this is the radius. When you flip them over, that is the same height on each point, so they

> bring them back together. The height would be the same because it is the radius of the triangle. That is per se because you have three triangles. (Richard, third interview)

While his interview showed that he had CCK required for the task, he had a difficulty in finding the relationship asked. After the rotation of internal triangles, the perimeter of the original triangle became the base of the new triangle. Rather than spatial reasoning on the animation, he understood that point, and therefore relationship from algebraic representation:

Interviewer:	What is the base of that big triangle now?
Richard:	I guess I am not sure what I am looking at.
Interviewer:	This is a new triangle after the animation. And this is the base of this new triangle. So what do you see there? How would you define that base? Could you define that with something that you know?
Richard:	No.
Interviewer:	OK. How would you represent or write down the base of that triangle, new triangle, in terms of letters?
Richard:	The length would be b, a, c.
Interviewer:	What is b, a, c for the original triangle? Are you adding them up?
Richard:	Yes. So b+a+c would be your length.
Interviewer:	Do you have a mathematical concept you can use for the sum?
Richard:	You are talking about the perimeter of the triangle. OK.

Through discussion with the interviewer, Richard showed how the area and perimeter of the triangle, and the radius were related. He seemed to understand the mathematics behind the animation, but conceptualized it more from the algebraic representation. In other words, he did not link the steady area of triangles to the translation of top vertices and their steady heights and bases. Because of these reasons, I coded that his SCK was not as much evident for this event as Karl and Kristin's SCK.

Richard considered GSP was helpful for his learning, especially because it allowed him to experiment with his ideas. GSP seemed to help him to create "what if" cases and to assess their validity for Euclidean geometry:

> I think one of the reasons why GSP helps because you can do a lot of what if. You can analyze it and you can do it very quickly. And like what we just went through... we can quickly redo it... (Richard, third interview)

Richard's view of GSP as a learning partner along with his CCK allowed him to demonstrate SCK. Overall, for this event, SCK has been demonstrated for PSTs who had CCK required, and who had a GSP view as a learning partner. Kathleen did not have CCK required and did not consider GSP conducive for her learning.

Pre and Post MKT-G Findings

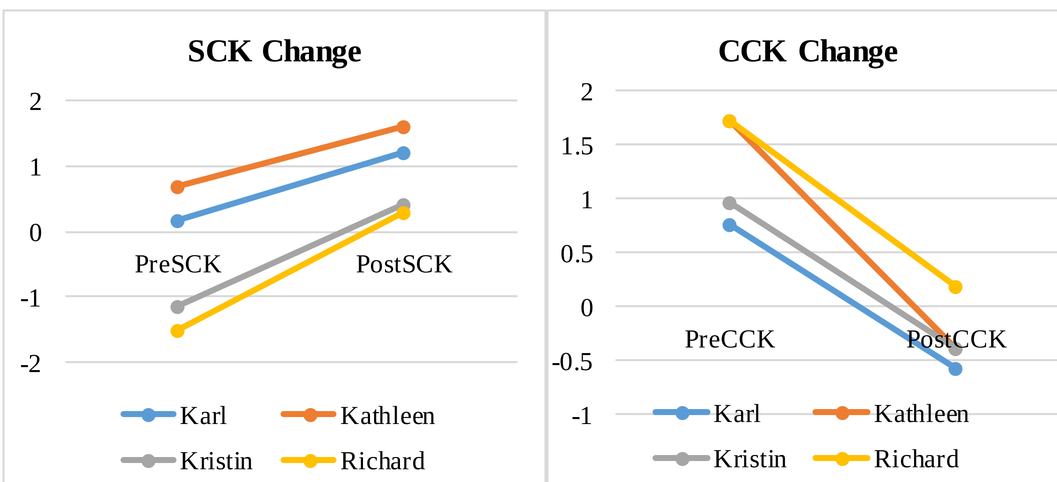

Figure 4.13: CCK and SCK Z-scores at Pre and Post MKT-G Assessment

Figure 4.13 represents four PSTs' SCK and CCK z-scores in the MKT-G assessment that was conducted at the beginning and at the end of the study. I included this figure here in order to understand how each one of the four PSTs' CCK and SCK were influenced while they received the same treatment. Since they experienced the same course throughout the semester, these differences might be attributed to their personal characteristics, preferences and views about GSP.

Eleven of the sixteen PSTs took both tests. Mean scores for SCK and CCK increased from pre- to post-test (56% to 57% for CCK, 44% to 54% for SCK). Only Kristin's CCK raw score increased from pre- to post-test, but CCK raw scores of the other three PSTs decreased. The four PSTs' SCK raw scores increased from pre- to post-test. Z-scores represented how each of four PSTs scored compared to other PSTs who took the test. Regarding the figure, each of four PSTs' CCK z-scores reduced from pre- to post-test. Four PSTs' SCK z-score increased from the pre to post MKT-G assessment. While Kristin and Richard scored lower than the average at the pre MKT-G assessment, their SCK was above average at the end of the study. Regarding this, among four PSTs, the most SCK change seemed to occur for Kristin and Richard.

The most decrease in CCK z-score seemed to occur for Kathleen. This finding means that four PSTs scored higher than the average of 11 PSTs for the CCK domain at the beginning of the study, they scored lower than the average at the end of the study. One possible claim is the geometry course in this study did not facilitate Kathleen's CCK development as much as her SCK development. In addition, it seems that the geometry course contributed to only Kristin's CCK considering that her CCK raw score increased from pre- to post-test. One of the major differences between Kathleen and Kristin throughout the semester was their views of GSP: while Kristin viewed GSP more as a learning partner, and therefore developed her CCK and SCK concurrently, Kathleen's view of GSP as a tool for efficiency could not allow her to develop the required CCK and SCK together. On the other hand, Kathleen's SCK score improved from pre- to post-test, which means the geometry course was facilitating her SCK development during the tasks where CCK was not a necessity. In the following section, I discuss the links among CCK, SCK, instructional decisions and views about GSP more in detail.

Discussion and Conclusion

SCK as a construct within Ball and her colleagues' MKT framework (Ball, Lubienski & Mewborn, 2001) is mathematical knowledge a mathematics teacher should possess as a subset of his/her subject-matter knowledge. In this study, I examined how teachers might develop SCK with GSP within a geometry content course and what factors and themes were emergent in SCK development for pre-service mathematics teachers. Four factors seemed to be influential for SCK development: 1) Common Content Knowledge, 2) Supportive Contexts and Instructional Decisions, 3) PSTs' openness to exploration; and 4) PSTs' views about GSP. Results showed that the availability of CCK was a priority for SCK development. Without having this prerequisite, PSTs' interaction with GSP did not always result in SCK development. After this requirement was satisfied, supportive contexts and instructional decisions such as providing opportunities for PSTs to justify their ideas, to explore their colleagues' errors or unusual procedures were also beneficial. The final two themes that were important in teachers' SCK development were PSTs' willingness to openly explore geometry independent of tools they use, and their predispositions and views about GSP. Each one of these factors is elaborated and discussed in the following sub-sections.

Common Content Knowledge

Data from interviews and class observations underlined the *necessity of CCK* in SCK development. Four focal participants managed to demonstrate SCK as long as they had the required CCK for the event. Simultaneous development of CCK enabled PSTs to demonstrate SCK during an event. For example, Kristin was not certain about the concept of external height for an obtuse triangle. Her CCK construction during the third interview enabled her to find the relationship for the inscribed circle of a triangle.

To triangulate this finding, I also examined participants' MKT-G results. Findings from the MKT-G assessment showed that four PSTs' SCK was developed, while their scores for CCK decreased. Regarding this, their experience within the geometry course contributed to their SCK, but somehow their experience did lower their CCK. In light of these results however, it is difficult to conclude that CCK is a pre-condition for SCK development. One possible explanation for the difference between qualitative and quantitative findings might be content-related. Whereas the content coverage for CCK and SCK was related and linked for the qualitative data, such relationship did not exist for the MKT-G assessment. Only 43% of items assessing SCK or CCK in the MKT-G assessment shared the same content. Examination of four participants in this study was another limitation in drawing a conclusion about the necessity of CCK for SCK development. Bair and Rich (2011) posited a similar claim about the need for CCK in SCK development. They discussed CCK might be considered as a requirement for SCK development, but found that PSTs having insufficient CCK still developed their SCK. However, they also pointed out that for higher level SCK attainment, CCK is a requirement. In this study, I did not differentiate if a task required higher level SCK or not, but the nature of the tasks was different. For example, the square construction was an open-ended task which required PSTs to explore geometry by themselves. The geo-board activity was also an open-ended task, as PSTs were asked to find multiple triangles having 2 square units area. The inscribed circle task was not an open-ended task as much as the other two tasks; it only required participants to interpret what they observed within the animation. Nonetheless, the content-level expected from participants was more complex for the inscribed circle task. Regarding this, one might consider SCK is higher-level for the inscribed-circle task, where CCK was a necessity more.

Supportive Contexts and Instructional Decisions

Cross-case analysis for Karl, Kathleen, Kristin and Richard pointed out commonalities and shared themes as well as the differences for the development of SCK within a technology-enhanced geometry class. Each PST's experiences throughout the semester during their class meetings and interviews indicated the importance of challenging their mathematical ideas and having an *Opportunity to Justify* them. Four PSTs' ideas on the given task with or without the GSP initiated an opportunity for them to develop their SCK. One of the reasons for this finding could come from the definition of SCK and its differentiation from CCK.

As defined in the theoretical framework, SCK is mathematical knowledge that a teacher is not supposed to transfer to students during the instruction, but uses in order to make sense of what mathematics is in action within students' mathematical errors, unusual procedures and personal definitions so that the teacher can utilize for the sake of students' learning. The teacher would use his/her CCK during the instruction by stating the axioms, facts, definitions, theorems, corollaries, and procedures to solve problems. Because SCK includes many possibilities depending on students' errors and unusual procedures, a teacher should encounter with these possibilities and reason on them. Regarding this condition, a challenge on ideas and procedures given by the instructor or by other PSTs and to justify those ideas would enable their SCK development.

Bair and Rich (2011) include a similar component in their framework for SCK development: *Explaining Their Reasoning*. In order to develop their SCK, teachers are expected to solve a problem first, explain their reasoning, and discuss possible students' errors. Ability to recognize possible students' errors and understand the mathematics embedded within these errors can be achieved with personal teacher experiences during which s/he made an error. The course instructor might notice these teachable moments, and a whole class discussion around these errors and/or unusual procedures to solve a problem might allow PSTs to develop SCK. In this study, the instructor tried to form a discussion around Kathleen's and other PSTs' procedures and mathematical errors in order to let them see how they mathematically reasoned, and what the mathematical source of the error or unusual procedure was. I assume that PSTs' recognition of their own and colleagues' errors and unusual procedures can be transferrable into their teaching when students make similar errors and share similar procedures.

Openness to Exploration and Viewing GSP as a Learner Partner

Throughout the semester, either the instructor of the course or PSTs used the GSP only for 30% of the instruction time. I also found that the instructor dedicated 70% of the instructional time for student-centered tasks that included whole class discussions, pair works, and individual work on open-ended geometry problems. With respect to these two percentages about the essence of the course, one might attribute the four focal participants' SCK development to pedagogical decisions taken by the instructor rather than the technology used. In light of this finding, PSTs' improvement of SCK scores from pre- to post-test can be linked with student-centered tasks and instructional skills of the instructor during the class meetings. However, these instructional skills and student-centered tasks could not contribute to PSTs' CCK scores. Moreover, the interview tasks showed that some teachers (e.g. Kathleen) could not develop SCK with GSP. This finding might also be attributed to the low percentage of the use of technology during class meetings. To be able to see the impact of GSP on PSTs' SCK development, having more opportunity to use technology during class meetings might be necessary.

This result supports Morris, Hiebert and Spitzer's findings (2009) that supportive contexts such as problem solving and discussions on PSTs' unusual mathematical procedures and/or errors are necessary for the development of SCK. GSP might be supportive for PSTs' SCK development because of its affordance to create an environment where they can explore principles for the geometry content, and test their ideas for generalization or refutation. If a PST, like Karl, is *Open to Exploration*, then s/he might construct SCK with or without the use of technology. On the other hand, if a PST, like Kathleen, Kristin or Richard, is not open to exploration and experimentation for mathematics, then the use of technology such as GSP might allow him to develop an explorative habit in geometry. For example, Kristin was explorative during 50% of the events (see Table 4.6) where GSP was not used, but she was explorative during 80% of the events (see Table 4.5 and 4.7) where GSP used.

Close examination of the eight events shared by Karl, Kathleen, Kristin and Richard unraveled the role of the technology, the GSP, in identifying conditions to strengthen the influence of the GSP in SCK development. Data indicated that *Views about GSP* seemed to determine whether GSP would be conducive for a PST's SCK development. If a PST views GSP as a tool for precise measurements and demonstration rather than a learning partner, then the role of GSP in SCK development might be limited. Kathleen did not view GSP as a learning partner during the third interview, and claimed her learning experience with GSP was painful, which might be the reason for her failure to develop SCK with GSP. On the other hand, Karl, Kristin and Richard considered the use of GSP advantageous for their learning, and they developed their SCK with GSP during this event.

Implications and Limitations

The major purpose of this case study was to explore the influence of electronic technologies on PSTs' SCK development through narratives and experiences of four participants within a content course. My qualitative analyses gave insight concerning the process of SCK development and for the discussion of possible factors affecting this process. While designing mathematics content courses offered for education majors, mathematics teacher educators might keep these factors in mind, implement instructional strategies emphasizing PSTs' reasoning around their errors and unusual procedures, form mathematical tasks requiring more exploration, and encourage PSTs to use technologies as learning partner rather than as new means for demonstration or measurement.

I am aware that findings from this study cannot be representative for every teacher educator or course setting. In order to increase validity of my findings, future research will focus on analyses of more teachers having different views about mathematics and technology. In addition, regarding the themes emerged from this study such as "openness to exploration" and "viewing the GSP as a learning partner", there might be a need for the investigation of PSTs' beliefs about mathematics and instructional technology.

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CHAPTER FIVE – TECHNOLOGICAL CONTENT KNOWLEDGE DEVELOPMENT

EXAMINING PRE-SERVICE MATHEMATICS TEACHERS' ACTIVE FORM OF TECHNOLOGICAL CONTENT KNOWLEDGE IN A GEOMETRY CONTENT COURSE

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Abstract

With the emergence of new instructional technologies for mathematics education, both prospective and practicing teachers are expected to develop principles of learning mathematics with technologies. As a result, many teacher education programs provide courses rich in technology experiences to pre-service mathematics teachers (PSTs) in order to equip them with required knowledge. Technological Pedagogical Content Knowledge (TPACK) (Koehler & Mishra, 2005) was one of the frameworks that identify the nature of knowledge required for technology integration in teaching content. One component of TPACK framework is Technological Content Knowledge (TCK), which was defined in this study as knowledge to use technological affordances while doing mathematics. This study examined 16 pre-service middle grade mathematics teachers' TCK development within a geometry course where they used Geometer's Sketchpad as the main instructional technology tool. This exploratory case study resulted in an analytical framework that can be used to assess PSTs' TCK development.

Keywords: Technological Content Knowledge, Pre-service Mathematics Education, Dynamic Geometry Software

Introduction

According to the *Professional Standards for Teaching Mathematics (PSTM)* by the National Council of Teachers of Mathematics (NCTM) (1989), prospective mathematics teachers were expected to know: mathematics and curriculum; students and their learning; and mathematical pedagogy. The *Principles and Standards for School Mathematics* (*PSSM*) (NCTM, 2000) in the following years depicted mathematics as a discipline connected to other subjects and to daily life. Regarding both *PSTM* (NCTM, 1989) and *PSSM* (NCTM, 2000), NCTM envisioned that high-quality mathematics instruction would be accessible to all students. In order for this vision to become a reality, mathematics teachers need to have a deep understanding of mathematical content as well as the ability to make pedagogical decisions that consider the diversified needs of a new generation of students. The development of *Common Core State Standards* (CCSSM) for mathematics (CCSSI, 2010) has accelerated the need for mathematics teachers who possess a strong mathematics background with deep conceptual understanding (Porter, Hwang & Yang, 2011).

Research has demonstrated that effective use of technology supports students' development of mathematical conceptual understanding (Mann, Shakeshaft, Becker, & Kottkamp, 1998; McCoy, 1996; Wiske, Franz & Breit, 2005; Roschelle, Shechtman, & Tatar, 2010). Computer tools can create opportunities for users to connect mathematical topics in a dynamic and interactive way. These tools also make the exploration of real life phenomena possible, allow learners to be exposed to central ideas, and create new mathematics (Cuoco, Benson, Kerins, Sword, & Waterman, 2010; Fey, Hollenbeck, & Wray, 2010).

Supporting students' development of a deep and conceptual understanding of mathematics is a difficult task (Eisenhart et al., 1993). This work can be further compounded when teachers do not also possess deep and conceptual understandings of mathematics themselves. PSTs' development of this knowledge can be supported through the use of technology within undergraduate mathematics courses, which can enable PSTs to develop an understanding of mathematical concepts (CBMS, 2001; Hollenbeck, Wray, & Fey, 2010; CBMS, 2012). The type of technology recommended for mathematics teachers to experience during their pre-service education included programming-based technologies such as C++, dynamic geometry software, and many more (CBMS, 2001; 2012).

Expectations for teachers to use technology effectively during instruction pushed scholars in the field of education to revisit the theoretical framework for teacher knowledge to explicitly address technology (Grangenett, 2008; Niess, 2008). As a result, a framework called *Technological Pedagogical Content Knowledge* (TPACK) emerged (Koehler & Mishra, 2005). According to the *TPACK* framework, mathematical content knowledge is evolving due to the infusion of technology. This new evolution of knowledge resulting from interacting with technology was termed Technological Content Knowledge (TCK) by Koehler and Mishra (2005). Regarding TCK, prospective mathematics teachers are expected to develop mathematical content knowledge as a result of interactions with the content through technology. For teachers to develop TCK, experience with technology was recommended within their teacher education programs (Niess, 2005; Lee & Hollebrands, 2008).

In spite of these recommendations, a review of the literature reveals mathematics methods courses have been often utilized as sites to aid the development of TCK instead of mathematics content courses, (Niess, 2008). And, the studies exploring TCK throughout teacher education programs did not address the influence of technology on PSTs' content knowledge, but focused instead on teachers' beliefs about their TCK (Koehler & Mishra, 2005; Niess, 2005; Lee & Hollebrands, 2008). Further, the review of the literature on TPACK demonstrated a need exists for further research investigating experiences that benefit or limit the development of PSTs' TCK (Bowers & Stephens, 2011). To address this gap in the literature, this study examined the experiences of middle school mathematics PSTs and how these experiences impacted their TCK development process within a geometry content course. Through this geometry course, dynamic geometry software (DGS) was used as the main instructional technology. This study tested the extent of the assumption that technology would influence the nature of geometry knowledge teachers construct. The following research questions guided my study:

1. How does a Technology Integrated Geometry Course influence pre-service middle school mathematics teachers' development of TCK?
2. Which factors influence the quality of pre-service middle school mathematics teachers' TCK in a Technology Integrated Geometry Course?

Theoretical Framework

I posited the manner in which PSTs use DGS, as a cognitive tool or merely as a new environment to draw geometrical models, might be an important factor for TCK development. As such, I begin by presenting the literature related to my study on the topics of cognitive tools and dynamic geometry software. As TCK was the main focus of this study, I next briefly describe the TPACK framework and its evolution in the field of education. I conclude this section with a discussion on how I employed the TPACK framework and the way I operationalized TCK for my data analysis.

Cognitive Tools

Cognitive tools are mental and computational devices that enhance and extend humans' thinking processes and cognitive capabilities, support knowledge construction, and release the cognitive burden through their expertise and possibility of intellectual partnership (Jonassen, 1992). Jonassen (1992) described cognitive tools as technologies that support learning through construction of knowledge and generative processing, the cognitive activity learners use to relate the incoming information to their previous knowledge with the use of cognitive tools.

Mayes (1992) discussed that cognitive tools have metacognitive and cognitive advantages. The use of cognitive tools develops users' metacognition by enabling them to learn skills about learning such as explanation, generation, argumentation or conjecturing and justifying. For example, the use of Geometer's Sketchpad (GSP) can teach students the necessity of reasoning and sense making during learning concepts and relationships in geometry.

Furthermore, cognitive tools create an intellectual partnership with users so that both the expertise of the tool and the user are shared and facilitated (Jonassen, Carr, & Yueh, 1998). The joint system of learning provides a cognitive advantage by off-loading unnecessary memorization tasks from user to computer. This cognitive release allows the user to be occupied with higher order thinking and deep cognitive processing skills (Kim & Reeves, 2007).

Specific advantages of cognitive tools for learners are: 1) engagement with higher order skills (Kim & Reeves, 2007); 2) durable encoding and retrieval of information (Mayes, 1992); 3) mind extensions (Jonassen, Carr & Yueh, 1998); and 4) externalizing representations. Meaningful learning opportunities given by the use of cognitive tools might be the reasons for these advantages. According to Jonassen, Carr and Yueh (1998), cognitive tools would work as mind extensions by doing unnecessary memory tasks for the user. The user does not need to utilize his/her cognitive capacities for computational or representational tasks, but understanding and connections of these tasks. Cognitive tools might also externalize conceptual representation of problems. Externalizing conceptual representation would enable the transfer of problem-solving skills from one domain to the other (Jonassen, 2003).

Although cognitive tools are innovations that bring expertise to the task and make the distribution of cognition possible, thinking, processing, understanding and interpreting are still the job of individuals, not of the computer. Cognitive tools are still unintelligent tools, which can create an environment for the user to use his/her intelligence in a better way (Jonassen, 1995). However, if a user approaches the tool from a traditional standpoint, and waits for the transmission of the knowledge from a computer rather than its construction; then the cognitive tool would not make a difference in the user's learning (Jonasses, 1992).

Cognitive tools are still design dependent. Technological affordances of cognitive tools as well as the quality of the accompanying task would make it advantageous in some settings, and disadvantageous in others. For example, a DGS might be very beneficial for students to understand construction of polygons as long as tasks and guidelines given with the technology make sense. On the other hand, the same technology might be confusing for students' understanding of irrational numbers because this kind of technology might be confusing to represent a line segment or a distance measured as an irrational number such as pi.

Considering a definition of cognitive tools that emphasizes knowledge construction (Jonassen, 1992), cognitive tools can best be supported by the use of constructivist instructional approaches. Cognitive tools cannot provide advantageous learning results when paired with traditional approaches such as drill, memorization and practice. Second, the quality of the task is as important as the affordances of the technology. Authentic mathematical tasks that demand higher-order thinking and deep processing skills would leverage the distribution of cognition between the user and the tool.

In summary, cognitive tools are technologies that facilitate PSTs' construction of knowledge. If used appropriately, these tools would also allow users to solve problems in a student-centered way. For this study, GSP was classified as a cognitive tool, and PSTs' construction of Technological Content Knowledge was examined with it.

Dynamic Geometry Software

Current mathematics education technologies include computers, computer software specific to mathematics (e.g. GSP, Tinkerplots or Fathom), graphing calculators, Smart Boards, and several other electronic utilities. In the middle or high school, it is expected students would be introduced to and become facile in using graphing calculators and a variety of computer software and mobile applications, both of which have potential to amplify students' mathematical conceptions. Dynamic geometry software (e.g. GSP or GeoGebra) is one type of instructional technology that is often implemented in classrooms as cognitive tools.

DGS is problem-solving tools that convert a static representation into a dynamic one. Its design features can encourage users to activate their cognitive processes (Santos-Trigo & Cristobal-Escalante, 2008). DGS also provides non-traditional ways to understand mathematical concepts by allowing users to see the relevancy of a statement through many visual examples in a few seconds (Marrades & Gutierraz, 2000). In addition, this type of software enables users to measure particular elements (e.g. sides or angles) of a geometrical object and to identify patterns and constants related to the object.

After DGS became prevalent in K-12 mathematics classes and was found to be effective in enhancing students' conceptual understanding of geometrical relationships, conjecturing and argumentation skills, teacher education programs began to include these technologies in the training of pre-service and in-service teachers. In other words, both practicing and prospective teachers are expected to know some type of specific mathematical technologies and methods to use them effectively in classroom. With the addition of this new knowledge base, the construct of teacher knowledge required modification to account for the emergence of new instructional technologies and related 21st century skills expected from teachers.

TPACK Framework

Teacher education programs initially focused on teachers' technology knowledge development through a techno-centric approach by which learning affordances and constraints of technologies became the basis for teacher preparation and further professional development. The techno-centric approach separated technology, content and pedagogy courses and did not enable teachers to integrate technology in a proper way. Harris, Mishra and Koehler (2009) identified the lack of content and pedagogy in this approach as its main weakness. In this approach, teachers might learn to use technology, but not learn how to integrate technology for specific mathematical and pedagogical goals (Harris et al., 2009). Koehler and Mishra's framework (2005) for Technological Pedagogical Content Knowledge (TPACK) was a reaction to the techno-centric approach, and provided an integrated knowledge structure that included technology. The TPACK framework became widely accepted as an appropriate model and approach for professional development that would organize activities and form opportunities for teacher learning within the consideration of the interconnection between specific content, pedagogy with technology. Its use was strengthened due to its focus on learners' deep conceptual understanding instead of drill and practice methods (Bowers &

Stephens, 2011). Researchers cited the importance of integrating technology knowledge with pedagogical knowledge and content knowledge, similar to the way in which Shulman (1986) proposed in his development of PCK (Koehler & Mishra, 2005; Niess, 2011).

The representation of the construct as well as the framework went through developmental processes: the first graphical representation included technology, content, learning and teaching as three intersecting sets, and their intersection indicated the TPACK. Koehler and Mishra (2005) extended these three sets into seven components in the handbook of Technological Pedagogical Content Knowledge (Figure 5.1): technological knowledge (TK), pedagogical knowledge (PK), content knowledge (CK), technological content knowledge (TCK), pedagogical content knowledge (PCK), technological pedagogical knowledge (TPK), and technological pedagogical content knowledge (TPACK).

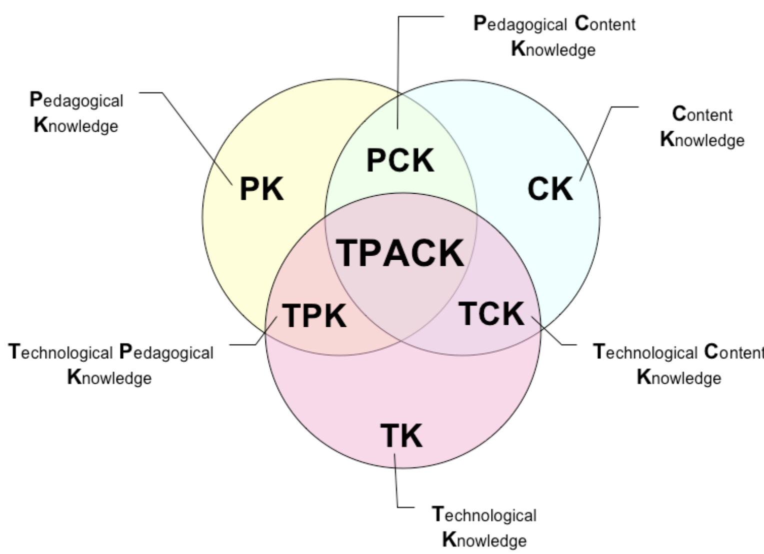

Figure 5.1: Representation of the TPACK Framework (Koehler & Mishra, 2005)

TPACK was considered as an appropriate framework to guide researchers and teacher educators for the preparation and evaluation of pre-service and in-service programs. Both Lee and Hollebrands' (2008) and Niess' (2005) studies showed this purpose and orientation. In Lee and Hollebrands' (2008) study, teachers practiced solving mathematical tasks with technology as learners, and later reflected on the capabilities and constraints of the technology they experienced in considering the use of the technology for teaching mathematics in future. In a way, teachers were guided to develop TCK through reflection. Niess (2005) also presented research from a content-based technology integration course. Within this course, teachers progressed through a program emphasizing research, problem-based and technology-integrated activities. Both Niess' (2005) and Lee and Hollebrands' (2008) studies enabled PSTs to restructure their TPACK by their learning experience with technologies. In Niess' study (2005), even though teachers had experience learning mathematics with technology before they started to teach, the study showed that their experiences were not sufficient for them to be aware of both affordances and limitations of the technologies in conceptually understanding mathematics.

Harris and his colleagues' (2009) articulated and clarified the constructs of the TPACK framework. According to their understanding, content knowledge consists of knowledge of concepts, theories, organizational frameworks, methods of evidence and proof. Pedagogical knowledge encompasses knowledge of educational purposes, goals, values, strategies, student learning and needs, classroom management, instructional planning, implementation and assessment. Their definition of technological knowledge was fluid and not as clear as content and pedagogical knowledge. This is largely needed as the definition needs to be able to change and adapt in light of the frequent development of new technologies. However, they did not restrict the notion of technology to Information Communication Technologies (ICT) or electronic technologies. For example, they also considered whiteboards as a technology used by teachers. Koehler and Mishra's (2005) definition of technology also encompassed commonplace technologies as overhead projectors, blackboards and books. Koehler and Mishra (2005) considered their definition of technological knowledge is close to that of Fluency of Information Technology (NRC, 1999), which was defined as broad understanding of information technologies to apply in everyday life and to assist when an informational technology is needed or not. As this study focuses on technological content knowledge, I unpack this aspect of the framework in more detail.

Technological content knowledge.

Technological Content Knowledge (TCK) "is useful to describe teachers' knowledge of how a subject matter is transformed with the application of technology" (Koehler and Mishra, 2005, p. 134). The definition of TCK assumes that both technology and content mutually influence and constrain each other. To show this two-directional interaction, Harris et al. (2009) gave examples from the developments in technologies and how these developments affected and enabled new discoveries in different disciplines. Grandgenett (2008) gave fractal geometry as an example of how school mathematics has been expanded with the use of technology.

Doerr and Zangor (2000) described two types of knowledge development for technological knowledge and technological content knowledge. Any user, teacher or student, would first interact with the tool, and this interaction would allow the user to make sense of the tool in terms of its capacity and limitations. The authors described this act as developing *meaning for the tool*. In a way, meaning for the tool supports the user's development of technological knowledge. The interaction with the technological tool would also facilitate understanding of concepts as well. During this process, the user would gain *meaning with the tool*. This experience would contribute to the development of users' technological content knowledge.

For this study, I operationalized TCK as teacher knowledge evident while the teacher either worked on a mathematical task with dynamic geometry software, or described his/her mathematical work as her statements of understood mathematical facts, procedures, or relationships after s/he used dynamic geometry software. In short, TCK was defined as teachers' mathematical content knowledge demonstrated behaviorally or verbally as a result of or in the presence of technology use. This definition was further refined to address two types of TCK. *Active form of TCK* describes teachers' *actions* utilizing dynamic geometry software in order to solve a mathematical problem or model a mathematical phenomenon. *Passive form of TCK* was evident in teacher statements where s/he verbally *interpreted* a mathematical phenomenon that was represented by dynamic geometry software. These two separate but supplementary definitions of TCK for my study allowed me to examine a geometry activity and differentiate the type of knowledge evident (Table 5.1). In this paper, I describe and analyze examples of the active form of TCK from class meetings in order to create a theoretical framework for TCK.

TCK Types	Description	Example
Active form of TCK	In a Type 1 TCK geometry task, teachers explore a geometrical phenomenon or solve a problem by using the affordances of the technology.	Constructing perpendicular bisector on GSP
Passive form of TCK	In a Type 2 TCK geometry task, teachers minimally use technology, and interpret the content demonstrated by the technology	Understanding a proof of the Pythagorean Theorem in front of a GSP animation.

Table 5.1: Active and Passive Form of TCK

Methods

Context

In this study, I examined PSTs enrolled in a technology-focused graduate geometry course, Geometry for the Middle Grades, seeking a Master's of Arts in Teaching at a Southeastern research university. The Master of Arts in Teaching (MAT) in middle level education is an accredited graduate degree program, which provides individuals who have received a bachelor's degree in another field an opportunity to transition to a teaching career in middle level education. I selected the Geometry for the Middle Grades course as my research site because one of the learning outcomes expected from PSTs enrolled in this course was to use dynamic geometric software, the GSP, flexibly and fluidly during problem-solving tasks. The GSP used for problem solving by PSTs allowed me to examine their Technological Content Knowledge (TCK) development and factors influencing this development.

The Geometry for the Middle Grades Course focused on understanding fundamental geometry topics pertaining to the middle grades curriculum through instructional methods such as learning by doing and constructing mathematical knowledge cooperatively. Learning goals for this course included: describing and understanding geometry content related to middle grades; connecting content to the Common Core State Standards for Mathematics (CCSSI, 2010); expertise in using dynamic geometry software for problem solving; creating inquiry-based geometry tasks to be used in middle school classrooms; and using and connecting multiple representations for geometry concepts. PSTs enrolled in this course did not have experience with the GSP at the beginning of the semester. They started to get used to the software through their personal endeavor with the GSP tasks given weekly.

The research data were collected during the implementation of this graduate course in fall 2013 semester. Students and the instructor met three hours a week, for a total of 13 weeks during the semester.

Participants

The instructor of the course is an assistant professor in the same university the study took place. She has been holding a joint appointment in the departments of mathematical sciences and teacher education since August 2012. She has her bachelor and master degrees in mathematical sciences, and her PhD degree in learning sciences. Before her graduate studies, she had been teaching secondary mathematics in urban settings for six years. Her teaching philosophy emphasized social constructivism and inquiry-based instruction. Her interviews after class observations indicated she favored PSTs to explore mathematical facts, terms and theorems collaboratively, and then to explain their findings to the whole class. She also viewed technology, especially GSP, as an opportunity for their exploration of geometrical phenomena and to understand geometry conceptually. When I was collecting data for this study, she was also the instructor of the methods course offered to pre-service middle grade mathematics teachers. In other words, a subset of PSTs participating in this study were taking both the methods and the geometry course.

In her teaching philosophy, the instructor emphasized the importance of inquiry-based teaching and learning, reflection, metacognition, assessment of PSTs' previous knowledge and their motivation for teaching and learning mathematics. The majority of her classroom activities were open to PSTs' different interpretations, analysis and conduct. Her typical instruction started with the presentation of an open-ended problem and a small whole-class discussion to engage PSTs. After that, she allowed PSTs to form a group to work on the presented problem collaboratively and to explore the mathematics within the problem. The final part of a typical instruction included an explanation phase by which PSTs presented their methods for the problem and discussed with others. The instructor deliberately chose these groups whose solutions involved errors in order to allow others to be aware of possible misconceptions their students might encounter in the future. She also used exit tickets at the end of almost each class meeting to help PSTs reflect on the mathematical focus of the class meeting, its connection to their previous knowledge, and its relationship to the real world. The instructor was also reflective on her teaching methods. During one of the class meetings, she asked PSTs to evaluate her instruction in terms of content, assessment techniques, and teaching methods. She stated during one of the interviews after the class meeting that she also requested their evaluation to help them be aware of teaching processes embedded within her instruction.

PSTs participating in this study were graduate students seeking teaching certification for middle grade mathematics education. They held bachelor degrees from different fields such as psychology, religion, business administration, economics, marketing, financial management, communication, electrical and computer engineering, nursing, physical sciences or engineering. All PSTs enrolled in the course (N=16) were invited and agreed to participate in the study at the start of the fall semester. Participants enrolled in the geometry course self-reported a lack of instructional technology experience or knowledge at the beginning of the semester. They also had not taken a geometry course or reviewed geometry content since high school. Of the 16 participants, only one participant had prior teaching experience.

Data Collection

Three types of data were collected during the semester: 1) an entrance survey; 2) non-participant observations, and 3) course artifacts (Yin, 2008). The collection of different types of data, such as surveys, documents and observations, allowed me to triangulate the data through multiple sources of evidence. This also created a chain of evidence, and trustworthiness for the findings from data.

An *Entrance Survey* was administered at the beginning of the study to each student enrolled in the course. It was designed to collect information about PSTs' background and establish their current beliefs about mathematics, teaching, and technology. A knowledge and belief profile for each participant was created with respect to his/her TCK level and responses to the belief-related questions from the entrance survey.

The second data source was a series of non-participant observations (Dewalt & Dewalt, 2002). According to the negotiation with the instructor, the author observed all class meetings from the beginning till the end of the semester. All observations except the first one were video recorded. In addition to the video recording, the first author took observation notes according to an observation protocol, which was designed to capture major events and issues occurring for PSTs while they were interacting with GSP, with another material or manipulative, and with the geometry content in order to understand the topic or their colleagues' strategies. After the observation of each class meeting, I created write-ups and contact summaries for each class observed (Miles & Huberman, 1994).

Course artifacts, such as Individual GSP Labs and Assignments, Group GSP Investigations, were also collected. While course artifacts were collected with respect to the negotiation with the instructor and permission given by the volunteers for the study, some of these artifacts such as Individual GSP Labs and Group GSP Investigations were prioritized according to their richness in order to answer my research questions.

During the Individual GSP Labs, students were guided to complete textbook-based prescriptive activities, which supported their exploration of middle grades geometry concepts. After they completed the activities, PSTs were required to upload a GSP document as their final product of their exploratory and/or problem-solving work. Throughout the semester, the instructor assigned seven Individual GSP Labs as homework or in-class activities. These Individual GSP Labs covered content such as the construction of a perpendicular bisector, the construction of an equilateral triangle, properties of parallel lines, similarity, and geometric transformations. In Table 5.2, I describe the prescriptive GSP tasks, when PSTs were assigned to work on during the semester, and what kind of GSP affordances they might have learned with these tasks:

Title of the Prescriptive GSP Tasks	**Date**	**Coverage of the Affordances within Tasks**
Constructing a Perpendicular Bisector	3 September 2013	Constructing a line, line segment, circle, and intersection point; measuring distances
Euclid's Proposition 1: An Equilateral Triangle	10 September 2013	Constructing a line segment, circle, intersection point, and triangle; measuring angles
Properties of Parallel Lines and Defining Triangles	17 September 2013	Constructing a line and parallel line; measuring angles and lengths
Similar Triangles	8 October 2013	Constructing a triangle, line, ray, parallel line, and perpendicular line; marking a point and angle as center and angle of rotation; rotating a line; measuring lengths; marking a line as a mirror
Geometric Transformation Tasks	22 October 2013	Constructing a polygon; marking a vector and angle; marking a line as a mirror; translating a line; rotating and reflecting the polygon; tracing points; showing grid; snapping points; measuring coordinates

Table 5.2: Prescriptive GSP Tasks

In addition to Individual GSP Labs, PSTs were expected to complete a set of GSP Group Investigations. In these open-ended tasks, PSTs were asked to solve a geometry problem or model a problem by using the GSP and present their findings with the whole class. I intentionally focused on these three tasks because their open-ended nature allowed me to examine PSTs' active form of TCK. The results section describes these tasks in detail.

Methodology and Data Analysis

A single case study with embedded units design was used as the methodology for this study (Yin, 2008). While the geometry course itself with its participants such as PSTs and the instructor of the course was the case of analysis, PSTs became the embedded units. Even though I investigated knowledge development and causal links between the knowledge development and factors affecting it, this study's main purpose was not to form statistical inferences, but to understand the phenomenon of knowledge development for teachers while immersed within a particular technology. In this respect, a qualitative study, more specifically a case study approach, was the most appropriate methodology to examine factors and their relationships of real-life phenomenon, within a specific context.

Write-ups from each class meeting were used to decide on important events and issues for my research agenda. I re-watched these video segments, and transcribed the discussion between PSTs and the instructor. I used only the video segments for the analysis during which PSTs used GSP for a class activity or assignment. The major purpose of the transcriptions was to narrate the events and issues. At this point, I also examined course artifacts such as lesson materials, participants' class works and assignments to elaborate on the details of the events.

I compiled data for all 16 participants' TCK development from class meetings and course artifacts. All compiled data were coded (Corbin & Strauss, 1998) to identify participants' TCK. To do so, I identified participants' statements and actions where their geometry content knowledge was determined. The codes, including a GSP integration, were considered as participants' TCK. I then de-identified each transcription by replacing participant’s statements during class meetings and their narrated actions with GSP with a pseudonym and the date when the statement or action took place. Table 5.3 demonstrates a representative data piece where I first coded her statement as evidence of CK (Content Knowledge), and more specifically as TCK (Technological Content Knowledge) because the participant explained her model for the shadow activity with GSP.

#	Date	Data Resource	Participant	Data	Knowledge Type
23	9.17	CM	Katherine	You can manipulate anything [in the shadow model] because I drew as a right angle. Any of these points change the height [of the light source and the object].	CK

Table 5.3: An Example from Compiled Data

After TCK segments and statements were identified for PSTs, I compared and contrasted these segments in terms of GSP affordances that they used. I grouped TCK representations of PSTs with respect to these affordances and identified them as patterns within the data. These patterns were then traced throughout the semester for each PST in order to understand if s/he added new affordances into his/her representations and therefore developed their TCK.

Results

In this section, my main goal is to present PSTs’ active form of TCK during three instructional tasks. After demonstrating PSTs’ different TCK for each task, the results section concludes with the presentation and application of an initial theoretical framework useful in categorizing participants’ levels of TCK.

Three tasks which I share data here are 1) perpendicular lines and shadow models, 2) pool pocket assignment, and 3) mirror madness assignment. Each of these three tasks was assigned to all PSTs. The PSTs were to utilize GSP in these tasks to model the given assignment dynamically, and if possible, to solve it by using its affordances. Each task took place in a different period of the semester (Table 5.4). I present them in chronological order.

Assignment Title	Date	Description of the Assignment
Perpendicular Lines and Shadow Models	17 September 2013	PSTs were asked to create a geometric model on GSP to demonstrate the relationship among the length of the shadow of an object, the height of a light source, the distance from the object to the light source, and the height of the object.
Pool Pocket Assignment	8 October 2013	PSTs were asked to investigate how the dimensions of a pool table effect the number of bounces that it takes for the ball to land in one of the pockets (if it does). On GSP, they were expected to draw pool tables with a different length and width, and the movement of the ball within the pool table they drew.
Mirror Madness Assignment	29 October 2013	PSTs were asked to find the height of four spiders from the given height of one spider and the distance between four mirrors through which each spider can see the other spider next to it. PSTs were guided to use GSP to model and solve the problem.

Table 5.4: Open-ended Assignment Descriptions

Perpendicular Lines and Shadow Models

The shadow models task was given as an in-class assignment during the third and fourth week of class. All participants were in groups of 3-4 people. During the third class meeting, PSTs in their groups were asked to experiment with different objects and light sources in order to examine how the shadow length of objects was changing with respect to factors they decided. This class meeting ended with the discussion of the independent variables that influence the length of the shadow. The instructor assigned all PSTs to create a dynamic GSP model of their experiment for the next class meeting. I considered this task as an opportunity to examine PSTs' active form of TCK because the task required PSTs to independently construct a model by using the affordances of GSP.

During the fourth class meeting, PSTs presented their GSP models for the Shadow Data Gathering group assignment. All PSTs, except Samuel and Katherine, constructed models that did not utilize perpendicular lines to have better dynamic models in picturing the shadow of an object under a light source. During the class activity, Samuel and Katherine were in the same group, but each created the GSP models individually as a homework. As a group, Erica, Cameron and Derek constructed a GSP model that consisted of lines and line segments. To make lines perpendicular to one another, they activated the grid form and use squares as their reference. In their model, they also measured the length of one of the line segments (Figure 5.2).

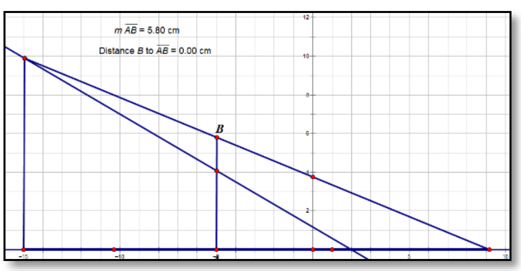

Figure 5.2: Erica, Cameron and Derek's Shadow Model

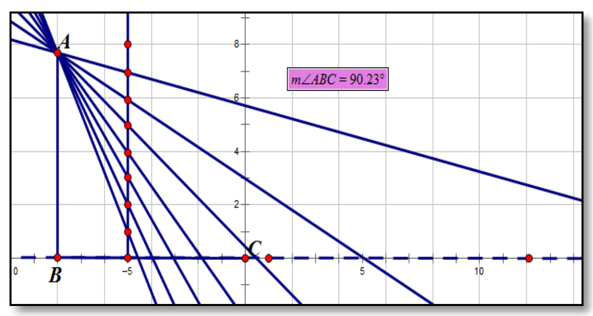

Figure 5.3: Victor, Laura, Karl, Kaci, Leonard's Model

Victor, Laura, Karl, Kaci and Leonard's shadow model was similar to Erica, Cameron and Derek's model (Figure 5.3). They also used the grid form of a coordinate system on GSP while constructing lines and line segments. However, these line segments were still not constructed as perpendicular line segments. To ascertain that one line segment is perpendicular to the other one, this group found the angle measure between two lines, which was approximately 90 degrees. Samuel and Katherine's models included different uses of GSP affordances. For example, in his model, Samuel constructed perpendicular lines and line segments to represent a light source, object and shadow (Figure 5.4).

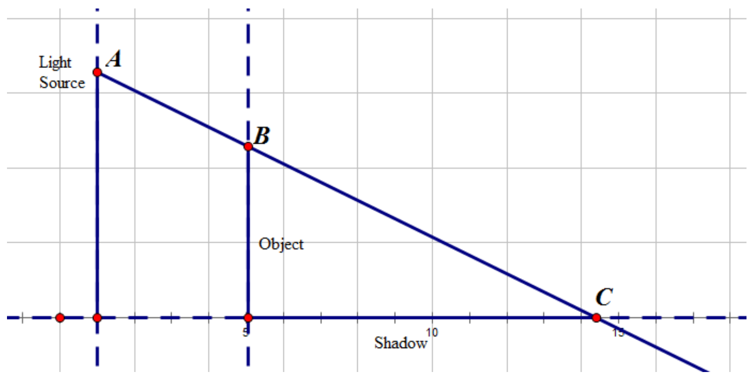

Figure 5.4: Samuel's Shadow Model on GSP

Because Samuel and Katherine used different features of GSP, the instructor of the course asked them to share their ways of model constructions:

Samuel:	What can you manipulate and what are your variables?
Katherine:	You can manipulate anything because I drew as a right angle. Any of these points change the height… the height of the… Although I got to still attach this [the line for the object was not connected to the projection line of the light from the light source] to itself.
Samuel:	What we did was we created a ray from … just from the source to the top of the object.
Katherine:	Because your ray keeps going.
Samuel:	Yeah. And then we put a point at the intersection of the ray with the ground.
The Instructor:	It sounds like there is issues with the order in which you constructed the parts of the model in order for it to preserve the feature, ray like…
Samuel:	Yeah. And you can select perpendicular line to make the right angle… We constructed a perpendicular line by… we put a straight line and we measured the slope by using the grid that you can make. [He showed the class how to find slope, and how GSP works for that purpose.] Then I formed a perpendicular line here [the source of light] and a perpendicular line here [the object]. Then have line segments so that have our guides. And then we had a ray from here to here [from the light source to the top of the object]. You can manipulate any of your different variances.

The dialogue between Samuel and Katherine demonstrated how they dealt with the right angle construction on GSP when needed. Even though both Samuel's and Katherine's models were not perfect in terms of their functionality for changes on each variable, and they accepted that there has to be a way to improve these models, their models were better constructions compared to the rest of the class. As Samuel pointed out, their model with the use of the ray feature allowed him to observe how the length of the shadow of the object was changing according to the change in the position of the light source or the height of the object:

Katherine:	Could you change the height of the source without changing the height of the object?
Samuel:	We can. One of the key things we had to do is, we had to do at the end, and we had to create a ray rather than the line segment or anything like that. So you drew one here to here [from light source to the top of the object], which allows you to do variations.

Right after this model presentation by Samuel and Katherine, other PSTs started to ask whether it is reasonable to do these constructions in order to construct a right triangle. Samuel and Katherine's methods in order to have a right triangle might have been seen as a challenge and complexity for these PSTs:

Jasmine:	What if I want to draw a right triangle? Do I have to do all of that?
Derek:	You can draw two lines, put a measurement for that angle… Go to measurement.
Cindy:	You can construct a perpendicular line onto segment. But I never could get… Like I tried to move that point.
Derek	I wish there was a button like right triangle, isosceles triangle, equilateral triangle…

Regarding this dialogue, PSTs approaches to this task were split; some used the angle measurement command and would drag line segments until they formed an angle whose measure is 90°. Other PSTs suggested using the construct perpendicular line command in order to do so. Derek's suggestion in this dialogue demonstrated at this point in the semester he was only capable of using the measurement tool to create perpendicular lines and had not yet differentiated between the ideas of "draw", in which once a figure has been drawn you do not move it and therefore it maintains its properties, and "construct"; in which the diagram maintains its designated properties, even when moved dynamically. In comparison, Cindy appeared to understand constructing perpendicular lines would create a 90° angle and recognized there is a GSP command to do so.

Pool Pocket and Mirror Madness

Pool Pocket Assignment was an individual assignment that was not designed to be solved with GSP originally. As in the "perpendicular lines and shadow models" activity, PSTs were first asked to solve the problem on paper, and then to use GSP in order to represent the given problem if they want to. The problem asked PSTs how dimensions of the pool table influence the number of the ball bounces around the table. The majority used affordances within their GSP sketches such as displaying grid, and drawing line segments in order to create pools in different width and length. To demonstrate the movement of the ball on the pool, PSTs preferred to use the diagonal of the square as a measurement tool for 45°angles. However, no PST used either perpendicular lines, or geometrical transformations to model the pool table or the movement of the ball.

Katherine's pool pocket model (Figure 5.5) included rectangles with different dimensions. In her sketch, Katherine also inserted text to answer the given problem. She recognized a pattern about the number of bounces in terms of the dimension of the pool table:

When height [of the pool table] is 2 [inches], if the width of the pool table is an odd number, then the number of shots is equal to the width plus 1 and the number of bounces is equal to the width; if the width of the pool table is an even number, then the number of shots is equal to the half of width. (Katherine, course artifact)

However, her model did not include a perpendicular line or geometrical transformation features in order to make the ball's bounce dependent on its location around the pool table. Katherine's model used the grid as a reference to draw rectangles and for the 45° angle of reflection of the bouncing ball.

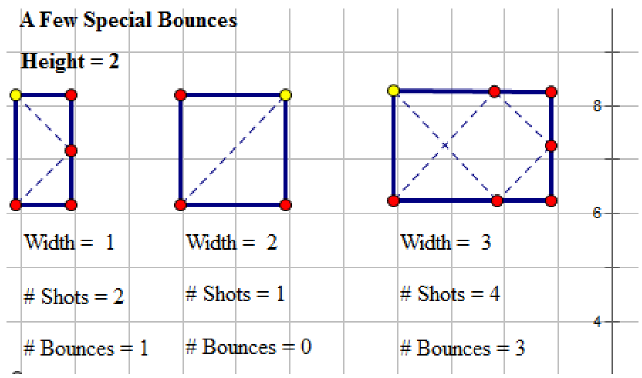

Figure 5.5: Katherine's Pool Pocket Model

Derek, Richard and Kristin's created their models individually, which were not all too different from Katherine's model (Figure 5.6). In their model, they also used the grid form and constructed polygons on the grid. I looked at their models by dragging the points on their GSP models, and recognized that all PSTs used GSP in order to draw rectangles with different shapes, but not to solve the given problem on GSP with one rectangle whose dimensions can dynamically change with dragging features.

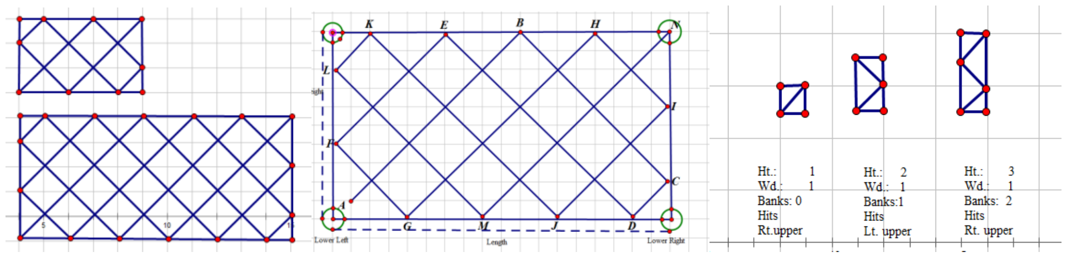

Figure 5.6: Derek, Richard and Kristin's Pool Pocket Models

PSTs used GSP as a drawing tool and did not integrate other features of the program to have a better model for this task. It seemed like PSTs used GSP as a graph paper with their drawings and notes they took on. Even though it was not a requirement for this task, I was expecting PSTs to explore the software more and create one rectangle to demonstrate the pool table where they can picture the shots and bounces of the ball for its different location by only dragging the point for the ball on the edge of pool table. To do that, PSTs needed to integrate geometrical transformation features of the software within their constructions.

The same as Pool Pocket Assignment, Mirror Madness was an open-ended, individual assignment. Although the task was not specifically designed for use with GSP, the instructor of the course asked PSTs to model and solve the given problem by using GSP and its affordances. As with earlier tasks, 15 of the 16 PSTs did not use GSP to construct perpendicular lines, but rather used angle measurements and made approximations. Kathleen was one of the PSTs who used angle measurement affordance of the program along with her approximations of the locations of spiders and mirrors.

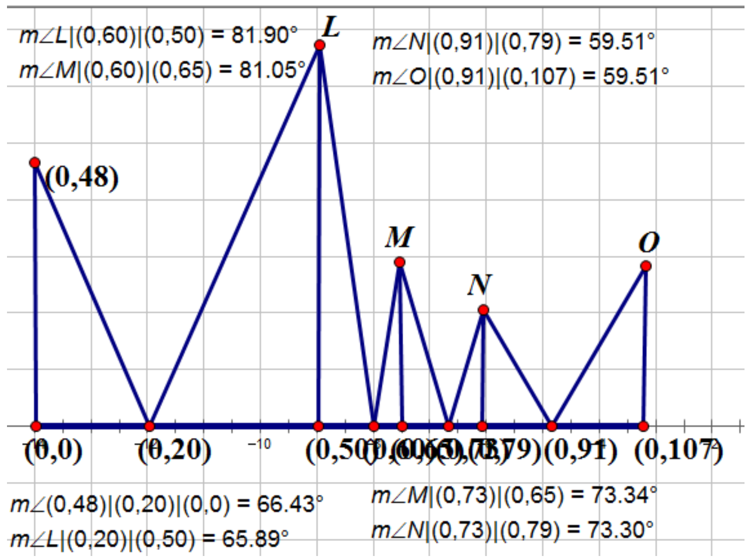

Figure 5.7: Kathleen's Construction for Mirror Madness

Kathleen did not use GSP as a tool to solve the given problem by using its affordances. Instead, she used the software as a technology-driven notebook where she could draw the model of the spiders and the mirrors without scaling it (Figure 5.7). In this model, points L, M, N and O stand for the locations of the spiders. Because the distance of the first spider and the distance between each mirror was given within the statement of the problem, she changed labels of these points into (x, y) coordinates. For example, the leftmost spider was labeled as (0, 48) because it was stated in the problem that this spider was 48 inches off the ground. Rather than scaling these distances on GSP, she made approximations for the locations of mirrors and spiders:

> I plotted my mirror and I thought how far the sister spider is going to be in the air. I have done the same for the other spiders with mirrors. I locked their positions. Then I put my line segments in all places. Because of that we were doing lately, we were looking at congruent angles about parallel lines and perpendicular lines. This is 90 degree and this is 90 degrees, these have to be the same angles

measured [the reflecting angles]. I graphed that point and drag it until I can make my angle manager as close to the same as possible and trying to maintain straight line. Although not perfectly exist, a little bit tricky. Then I made my estimation over here with the pen tool. I added distances to the point, point mirror, mirror point… So I was like "now I have to prove it mathematically"… I decided to go to the proportions… That is why I wrote up that way to get to the mother. I was looking at height and width, but I did not know how to do it. So, I set up that relationship. (Kathleen, class observation)

Cindy's construction for the same assignment was quite different from Kathleen's construction. Cindy first evaluated the reliability and preciseness of GSP for her findings, and described one of the limitations she figured out from her construction and interaction with GSP for this assignment:

> Because I was not thinking proportional triangles first, I thought that the angle that hits mirror is going to be the same angle going up the mirror. When I constructed the thing on GSP, the height was 48. The bottom was 20. I measured the angle, and it came out to be 67.38. Just to check it, I checked it with trigonometry. Yes in fact, it was 67.38. GSP measured that correctly. Then I used transform translate. And translate a point to the angle 67.38 degree, and intersected that with the line that went through 30 away… With GSP, then you can measure that height, comes up with 71.72. Even though these two angles were exactly correct, and the bottom was exactly correct, when those two lines intersected, there were some sort of built in… error. (Cindy, class observation)

Regarding her description above, she used geometrical transformation as one of the GSP affordances for her construction. After she found what she called a "built-in error" of GSP, she evaluated the capacity of the software with a second construction she made. Her finding of GSP's built-in limitation seemed to push her to explore the software more in order to understand how it worked for the Mirror Madness Assignment:

> But when you used the coordinates like zero as the starting point, then it comes up perfectly. If you do measure distance by coordinate distances. Everything works out perfectly. (Cindy, class observation)

Cindy's second construction included plotted points on the coordinate plane of GSP (Figure 5.8). Instead of approximating the locations of the spiders and mirrors, Cindy managed to locate each element of the problem in a precise scale, so that she did not need to make further calculations on a piece of paper in order to find the height of the other spiders.

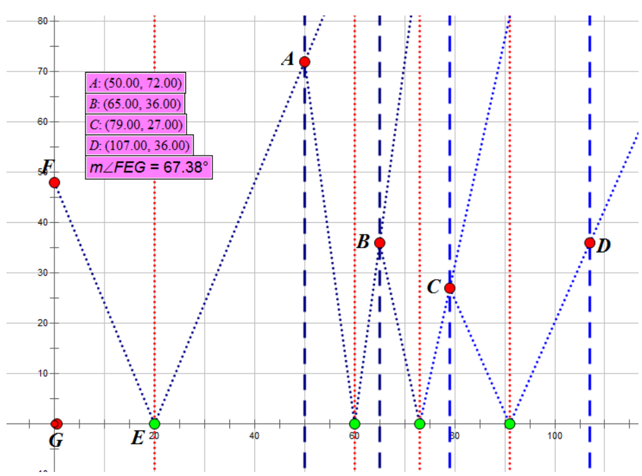

Figure 5.8: Cindy's Construction for Mirror Madness

Cindy’s both the first and second constructions on GSP were novel regarding her precise representation of the problem as well as her reasoning process about the limitation versus capacity of GSP for a specific geometry problem. I considered her work as a higher level TCK because of the compactness of her content and technology knowledge and their interaction for the demonstration and solution of the problem in a different medium. Her effort of using different mathematical methods such as proportionality and trigonometry indicated her sound content knowledge. In addition, she explored different affordances of the software as geometrical transformations to rotate a line segment, and to plot points for the benefit of having a scaled model. Overall, her TCK for this task was considered as a stronger example than what Kathleen constructed.

Theoretical Framework to Assess PSTs’ TCK

The quality of the 16 participants’ active form of TCK during three tasks were compared and contrasted, and a theoretical framework consisting of four levels was created accordingly (Table 5.5). Even though I used data from PSTs in this study, the framework could potentially be utilized to assess any novice users, including in-service teachers, of GSP.

While forming the levels, I focused on the complexity of the technological affordances used by participants in order to solve a geometry problem, or model a geometrical phenomenon. Returning back to Koehler and Mishra’s definition (2005), TCK is the subject-matter knowledge that was transformed with the application of technology. I assumed that an understanding of the different affordances of technology would change the subject-matter in which PSTs engaged. In other words, a teacher would have a higher level TCK as long as s/he knows complex affordances of the technology and integrates them in order to transfer the subject-matter.

As these levels emerged from the data collected in an exploratory study, I do not suggest that level 4 would be the highest level a mathematics teacher could reach, only that this was the highest level of TCK exhibited by these participants. Depending on teachers' content knowledge and their comfort with the technology, there could be higher levels of TCK a teacher would reach with the use of GSP. After unpacking the four levels evident in this study, I present one more level (level 5) as a part of a hypothetical learning trajectory (Jackiw, 1997).

TCK Level	**Description of the Level**
Level 1 Superficial TCK	Uses only affordances of GSP that are available in the toolbox to model a phenomenon or solve a geometry problem on GSP. Creates points, lines, circles, polygons and text to draw figures.
Level 2 Isolated TCK	Can use measurement affordances to model a phenomenon or solve a geometry problem on GSP. Measures angles and length of line segments, activates grids and coordinate plane for drawings, and/or use slope feature in order to confirm propositions. Affordances are used in isolation.
Level 3 Preserved TCK	Can use construction tools to model a phenomenon or solve a geometry problem on GSP. Constructs parallel lines, perpendicular lines and utilizes geometrical transformations in order to make constructions rigid. Features of the geometrical figures constructed are preserved.
Level 4 Integrated TCK	Can use and integrate different features of the program in order to scale his/her constructions while solving a problem or modeling it appropriately. With the use of such affordance, PST pictures his geometry knowledge in a holistic way. Several affordances of the technology are used in integration rather than in isolation. While PST creates dynamic model if the problem asks him/her to explore, s/he sets and preserves the given measurements as stated in the problem.
Level 5 Expert TCK	Can use complex affordances such as creating their own custom tools or animations depending on students' needs and instructional learning objectives for complex geometrical topics which require comparison of different axiomatic systems.

Table 5.5: TCK Levels to categorize PSTs' Active form of TCK

Level 1. There was no data from three tasks presented which contained a representative example for level 1 TCK. Although I did not observe evidence of these characteristics specifically, I hypothesized there has to be an entrance level of TCK, which can be defined with the use of affordances available with the tools within the toolbox. Utilizing these tools, a PST can create points, lines, line segments, circles and polygons, and insert text when needed. At this level of TCK, the PST approaches the tool as a new medium to draw geometrical figures on a computer, but nothing else.

Level 2. The analysis of the data from the tasks presented indicated many PSTs often were limited to use of the measurement affordances of GSP as they modeled and solved the tasks presented. Derek’s suggestion to draw a right triangle was an example for this level of TCK. He recommended another PST to use the angle measurement tool in order to create a 90° angle. On GSP, a PST needs to use different tools under the menu bar; measurement tools are one of these tools. Similar tools that can be accessed under menu bar are graph menu to activate grid form or define the coordinate system, measuring slope under the measurement menu. PSTs in this study used grid form to draw perpendicular lines, or measure the slope for the same purpose. Regarding these examples from the data, I categorized the use of these types of affordances under the same level of TCK: level 2. PSTs with level 2 TCK use GSP as a drawing tool and measurement tool, not for constructions. Even though PSTs might construct some geometrical figures, the quality of these constructions was not adequate because the features they targeted to be maintained with their constructions were not preserved dynamically. At this level of TCK, teacher uses affordances for measurement in isolation, but not in integration.

Level 3. A higher level of TCK was achieved when PSTs started to utilize features of the construction menu to construct parallel lines, perpendicular lines and/or geometrical transformations. By using these affordances, PSTs models would be more mathematically precise as the construction feature allows PSTs to preserve the geometrical features targeted. Katherine and Samuel's shadow models represented this level of TCK. Rather than using measurement tools or activating the grid form, they utilized the constructing perpendicular lines command in their constructions of the shadow models. By using this affordance, their constructions were preserved when any point on their models was dragged. I also considered the use of geometrical transformation affordances under this level of TCK. Teachers at this level of TCK would use geometrical transformations to preserve the features of a geometrical figure they constructed. In their constructions, they do not intend to use geometrical transformations in isolation, but to have better models in integration. However, the integration of the affordances is not as much as in the next level. Because the emphasis is on the preservation of features of geometrical figures, this level was called Preserved TCK. My observations and data revealed limited examples of the use of this type of affordance. Cindy was the only PST who used this affordance while constructing the model for mirror madness problem. Potentially, PSTs could also have used the rotational transformation affordance of GSP to model the pool pocket problem, but no PSTs constructed their models in that way.

Level 4. The next level of TCK represents PSTs' ability to use multiple features of GSP in concert to solve problems, to map the given problem onto the software by preserving the given measurements, and to use its dynamic features for the rest of problem requiring exploration. In this level of TCK, teachers are aware of many technological affordances, and also use them in integration while constructing a geometrical model to solve a problem. In other words, teachers do not only construct scaled models, but also use other affordances of the technology in integration. In addition to the use of features such as parallel/perpendicular lines or geometrical transformations, I expected PSTs at this level to scale their models with respect to the given information in the problem. For example, the mirror madness problem specified the distance between each spider and the height of the first spider from the ground. In a scaled model, I expected PSTs to use this information on the coordinate plane by plotting the points. Cindy's model for the mirror madness problem was such an example from the data.

For the pool pocket problem, the given features of the problem are side lengths of the pool table in unit measures and movement of the ball. Since the question is asking to observe how the number of bounces is changing as the side lengths are changing, the rectangle should be dynamic so that the user can change the side lengths. While you are changing the side lengths of the rectangle, the movement of the ball has to be preserved because it is following a geometrical rule (i.e. reflection) while moving. At that point, the degree of angle for rotation has to be scaled to 45 degrees.

Level 5. PSTs would create their own tools for easier geometrical constructions, animations or operate on other affordances that require a high level of expertise with GSP in order to help students understand complex topics in geometry, which might even be in the scope of Non-Euclidean geometry. For example, the developer of the software describes (Jackiw, 1997) how hyperbolic geometry can be embedded in the Geometer's Sketchpad, and that way, students might compare the relevance of Euclidean axioms on different planes.

During one of the class meetings, Derek stated an expectation he had of GSP would be a command he could use to draw a right triangle automatically. Although the instructor of the course introduced and discussed the custom tool affordance of GSP as a way to meet this expectation, none of the participants practiced or utilized this affordance during the semester. PSTs also observed how animations would operate on GSP, but they were not instructed in how to create one. As a hypothetical learning trajectory, the next level of TCK would require teachers to use technology and content together in order to create instructional products both as a technology and content expert.

The hypothetical learning trajectory presented in the Assessment of Teacher's TCK Framework progresses from a superficial and isolated usage and demonstration of content with technology (level 1) to a coordinated and integrated usage and demonstration (level 5). Teachers develop their TCK from a superficial content and technology user to a more expert content and technology user.

Table 5.6 presents participants' level of TCK with respect to the theoretical framework developed from this data set. Data for the shadow model revealed that majority of participants (13 of 15) exhibited at best, level 2 TCK. Only 3 participants (Cindy, Katherine and Samuel) used the construct menu when creating right angles, thus demonstrating level 3 TCK. There was no prescriptive GSP assignment that focused on how to construct perpendicular and/or parallel lines until after the shadow data activity.

Even so, some PSTs (Cindy, Katherine and Samuel) seemed to explore the software by themselves and became aware of this affordance. Other PSTs used lower level affordances when they needed to create a right angle. Instead of using perpendicular lines, they preferred to use angle measurements to be sure that the angle between two lines or line segments were 90 degrees.

Participants	Shadow Model	Pool Pocket Problem	Mirror Madness Problem	Highest Level Attained
Kristin	Level 2	Level 2	Level 2	2
Derek	Level 2	Level 2	Level 2	2
Cameron	Level 2	Level 2	Level 2	2
Victor	Level 2	Level 2	Level 2	2
Kathleen	Level 2	Level 2	Level 2	2
Leonard	Level 2	Level 2	Level 2	2
Richard	Level 2	Level 2	Level 2	2
Katherine	Level 3	Level 2	Level 2	3
Cindy	Level 3	Level 2	Level 4	4
Karl	Level 2	Level 2	Level 2	2
Abby	Level 2	Level 2	Level 2	2
Erica	Level 2	Level 2	Level 2	2
Samuel	Level 3	Level 2	Level 2	3
Kaci	Level 2	Level 2	Level 2	2
Laura	Level 2	Level 2	Level 2	2
Jasmine	Level 2	Level 2	Level 2	2

Table 5.6: The Distribution of PSTs in terms of their TCK Level

PSTs' constructions for the pool pocket and mirror madness problems did not show evidence for their improvement of TCK into level 3 or 4. Even though three PSTs were aware of constructing a perpendicular line and their previous constructions included these features, they did not use the same affordances in the pool pocket problem. For the mirror madness problem, only Cindy's construction showed evidence for level 4 TCK. While Kathleen's work on GSP for the mirror madness problem was categorized with level 2 TCK the same as the other 14 other PSTs, Cindy's construction for the same problem was identified with level 4 TCK. In this table, I also included a column to demonstrate their highest level of TCK they reached throughout the semester. I considered that the highest level would represent the PST's potential and her TCK attainment through his/her experiences within the course in one semester.

Discussion and Conclusion

PSTs' experiences within the geometry course revealed differences in their TCK development throughout the semester. Some prospective teachers learned to use the various affordances of GSP and understood the geometry embedded within the GSP environment in a better way. However, other teachers did not develop into the same level of TCK as their colleagues. In the last part of the results section, my research data demonstrated all teachers demonstrated level 2 TCK at some point; some of them demonstrated level 3 TCK as their highest level of TCK they reached. One PST reached level 4 TCK at one of the tasks. These findings show that PSTs mostly represent level 2 TCK, which emphasizes the use of measurement affordances that are accessible from its menu bar. GSP was a new technology for these PSTs and they only had experience with this instructional technology for one semester, which might not have been sufficient time to learn its capabilities. Apparently, the majority of PSTs in this study used GSP as a technology replacement for other tools such as pencil/paper and/or calculator. Kathleen's use of GSP as a notebook page in the pool pocket problem is one such example. The PSTs might not have viewed or used GSP as a technology that they can solve problems with, but only as a new platform to draw the sketch and make precise measurements. Learning to use GSP as a technology to construct mathematical models might require more time and experience.

Viewing and using GSP as a problem-solving tool might also create a challenge for PSTs. Rather than dealing with such a challenge, PSTs might have chosen to work with GSP according to the guidelines written in the task. For example, in the pool pocket problem, PSTs were guided to form squares and rectangles with different dimensions, not to form a dynamic polygon with dimensions that could be modified by dragging. To create such a construction might be a challenge that requires more content and technology knowledge.

The prescriptive GSP assignments were given to and done by all PSTs. Regarding these prescriptive assignments, I would expect PSTs to become familiar with GSP affordances such as constructing perpendicular/parallel lines or using geometrical transformations. However, this was not observed in the majority of the PSTs. Two PSTs reached level 3, and one PST reached level 4 TCK. These PSTs' higher TCK might be attributed to their personal interest and endeavor in learning GSP more. Having a deeper mathematical knowledge might have guided these PSTs to utilize GSP more than other PSTs. Regarding Cindy's case, one might hypothesize that PSTs' active form of TCK can develop more if they have possess a deeper understanding of mathematical knowledge. Guerrero's work (2010) supported this hypothesis and discussed that using technology to explore geometry in depth would present teachers with different mathematics content, which results in an expectation from teachers to be more confident in their content knowledge. Cindy also used GSP in order to solve the given mirror madness problem. In a way, she built an *intellectual partnership* with the software. This use of GSP by Cindy was more aligned with the definition of cognitive tools (Jonassen, 1992).

PSTs' comfort with GSP might have been another factor affecting their TCK development. Each teacher devised his/her own way to overcome a problem depending on his/her comfort with the GSP. During one of the class meetings before the first interview, Derek suggested his classmates to use the angle measurement affordance and dragging feature in forming a right angle. Even though Cindy responded to Derek that he might also use the perpendicular line feature for the same purpose, he preferred to use grids as a way to form rectangles rather than perpendicular lines for the pool pocket problem. Several PSTs used the same affordance during the pool pocket problem.

Beyond being able to assess and classify PSTs' TCK levels, teacher educators must also seek to motivate progress toward the use of technological affordances in integration. I propose the following actions to take into consideration by teacher educators to support teachers' TCK development from superficial TCK into expert TCK. To begin, PSTs should be provided with opportunities to work on "authentic curriculum problems for which technology-based solutions are collaboratively designed" (Voogt, Fisser, Roblin, Tondeur, & van Braak, 2013, p. 118). By presenting PSTs with authentic problems, they would perceive a purpose and necessity of the use of technology. The problems proposed should not be solved on paper, and should necessitate the use of technology because of their complexity.

Further, Koehler and Mishra's (2005) introduction of "learning technology by design" seems to be an effective way in order for teachers to develop their TCK. The teacher educator of the course that I investigated allowed PSTs to work on authentic geometry problems, and the instructor asked them to use GSP to model these problems.

However, I considered that guiding PSTs to use GSP in order only to represent a problem is not sufficient for “learning technology by design”. In order to accelerate teachers’ “learning technology by design”, PSTs should be introduced to use technologies as problem-solving tools as much as a new environment to represent a model. This approach would also help teachers shift their perspective from seeing GSP as a measurement tool to more of a tool for exploration and problem solving.

The teacher educator should also underline the identification of misconceptions, and seek technological representations in order to overcome these problems (Akkoç, 2011). Such an approach would both support teachers’ content and TCK development at the same time. In this study, PSTs encountered with some errors while constructing their models on GSP. Close examination of these technological errors and encouraging teachers to find the reason for their error and eliminate it would allow them to learn new technological affordances, preserve the features of their models, and use these affordances in integration more.

In this study, I examined TCK development of PSTs who changed their career from different fields into teaching. Because of that, my participants’ background, previous experiences with mathematics, technology and pedagogy became one of the intervening factors for their development. These factors that I discussed for TCK development might not be applicable for PSTs who have a stronger mathematics preparation. Future research would investigate TCK of PSTs who majored in mathematics and/or mathematics education to see how the levels of TCK might be altered or modified.

Finally, although I focused on exploring rather than explaining PSTs' TCK development, this study allowed me to create an analytical framework useful for assessing PSTs' TCK development. The framework might be used by teacher educators in setting technological content goals along with content goals for their content courses, as well as used for future research in identifying PSTs' TCK and how its development was promoted or hindered in different course settings. This framework is not intended to be an end product, but an initial classification that can be reorganized, restructured, or extended with respect to further research with the affordances of a technology covered during a geometry content course.

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CHAPTER SIX – INVESTIGATING BELIEFS ABOUT MATHEMATICS, TEACHING AND TECHNOLOGY

A TALE OF FIVE PRE-SERVICE MATHEMATICS TEACHERS: INVESTIGATING BELIEFS ABOUT MATHEMATICS, TEACHING AND TECHNOLOGY IN THE DEVELOPMENT OF GEOMETRY KNOWLEDGE WITH GEOMETER'S SKETCHPAD

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Abstract

The influence of instructional technologies on a teacher's subject-matter knowledge development depends on their background, experiences and beliefs about mathematics, teaching and technology. Departing from this hypothesis, this study examined how pre-service middle grade mathematics teachers' beliefs about mathematics, teaching and technology are related to the development of their subject-matter knowledge specific to the teaching profession (Specialized Content Knowledge). While Ball and her colleagues' Mathematical Knowledge for Teaching was used for the assessment of SCK, three different theoretical frameworks were used to categorize participants' beliefs. The multiple case study analysis revealed that pre-service teachers' different beliefs about mathematics and technology indicated a different impact of the use of Geometer's Sketchpad on their SCK.

Keywords: Mathematical Knowledge for Teaching, Specialized Content Knowledge, Teacher Beliefs, Instructional Technologies, Dynamic Geometry Software

Introduction

For many years, teacher education programs adopted a transmission model, which prioritized content knowledge acquisition as the only requirement for teachers to teach (Darling-Hammond, 2006). However, findings from research (Monk, 1994; Rivkin, Hanushek, & Kain, 2005; Ball, Hill, & Bass, 2005) in the last two decades demonstrated knowing mathematics was not sufficient for teachers to teach mathematics effectively. Even though teacher education programs emphasized gains in pedagogical knowledge in addition to mathematical content knowledge, advanced mathematics courses still dominated pre-service programs and coursework.

The MET II report (CBMS, 2012) was the first report that recommended prospective mathematics teachers to take mathematics courses offered only for mathematics teachers. These mathematics course revisited concepts in different branches of mathematics with an emphasis on deep understanding, which can be described as knowing more than simply how to "do" mathematics. Ball and her colleagues (2001) called this subject-matter knowledge with an emphasis on deep understanding and germane specifically to the work of teaching mathematics as Specialized Content Knowledge (SCK).

A narrowing of focus to literature addressing pre-service mathematics teachers' development of SCK produced limited results. One reason for this limitation could be the difficulty in differentiating SCK from other types of subject-matter knowledge or from aspects of Pedagogical Content Knowledge (Speer & Wagner, 2009). Conceptualization of *SCK* within the *MKT* framework served as evidence for the importance of building knowledge for changes projected by reform-based standards (Goertz, 2010). This conceptualization also increased the challenge concerning how such knowledge development through teacher education would be accomplished. One possible way to accelerate teachers' SCK development is the use of electronic technologies. Research has demonstrated that effective use of technology supports students' development of conceptual understanding (Mann, Shakeshaft, Becker, & Kottkamp, 1998; McCoy, 1996; Wiske, Franz & Breit, 2005; Roschelle, Shechtman, & Tatar, 2010). The same premise may hold true for teachers' SCK development when instructional technologies are used effectively.

Due to the availability of dynamic visualization and fast information access, electronic technologies make the exploration of real life phenomena possible, allow learners to be exposed to central ideas, and create new mathematics to learn (Cuoco, Benson, Kerins, Sword, & Waterman, 2010; Fey, Hollenbeck, & Wray, 2010). The use of virtual manipulatives provides teachers an opportunity to experiment with geometry and question the validity of theorems for different conditions (Hollenbeck, Wray, & Fey, 2010). Such theorems may be discovered through paper and pencil, but the use of dynamic geometry software would enable a more time-efficient discovery. The literature examining the influence of technology on SCK development (Silverman, 2012; Silverman & Clay, 2009) did not specifically focus on SCK development, but looked at MKT in general. In light of the limited literature, I focused my study on the impact of dynamic geometry software on pre-service mathematics teachers' SCK development.

Since my intent was to closely examine the SCK development as a phenomenon, I also hypothesized that teacher beliefs would be a factor mediating this phenomenon.

Beliefs can also influence a person potential to gain from an environment and therefore to develop content knowledge. For example, a pre-service teacher who has positive beliefs about the use of technology for mathematics instruction might have a better chance to learn the content with technology. On the other hand, another pre-service teacher might have difficulty in learning content with technology because s/he does not consider technology as a facilitator for his/her learning. Teachers' beliefs about the nature of mathematics or better instructional techniques may also influence their learning gains in a mathematics content course.

While there are studies looking for the effect of beliefs on teachers' practice (Borko, Mayfield, Marion, Flexer, & Cumbo, 1997; Guskey, 2002), research on the relationship between teachers' beliefs and their knowledge development is rare. Existing research focused on teachers' PCK development and how their beliefs about teaching are related to the development (Hollingsworth, 1989). In addition, the research did not make an attempt to clearly differentiate beliefs from content knowledge.

Regarding the gap in the literature about the use of dynamic geometry software and the quality of geometry content courses for PSTs' SCK development, I examined the experiences of pre-service mathematics teachers and how these experiences impact their SCK development process and beliefs about mathematics, teaching and technology within a geometry course. Secondly, I examined the link between teachers' beliefs and their content knowledge development throughout the semester. I hypothesized technology would influence the nature of mathematics they learn and the development of mathematical content knowledge specific to teaching (Ball, Thames, & Phelps, 2008). The study tested the extent of these assumptions. The following research questions guided my study.

1. How are pre-service mathematics teachers' beliefs about the discipline of mathematics related to their SCK development?
2. How are pre-service mathematics teachers' beliefs about teaching mathematics related to their SCK development?
3. How are pre-service mathematics teachers' beliefs about technology related to their SCK development?

Theoretical Framework

Defining Knowledge as a Construct

For many scholars, knowledge is a cognitive construct hard to clearly define and describe. The difficulty especially arises from its differentiation from beliefs. Verloop, Driel and Meijer (2001) differentiated knowledge from beliefs by defining the former "as an overarching, inclusive concept, summarizing a large variety of cognitions, from conscious and well-balanced opinions to unconscious and un-reflected intuitions" (p. 6), whose correctness was confirmed in our minds.

Lemos (2007) discussed the correctness of information according to its correspondence to the negotiated facts. According to Lemos (2007), there are three types of knowledge: 1) *how to* knowledge, 2) acquaintance knowledge, and 3) propositional knowledge, which consists of facts and true propositions. A proposition can be considered true if and only if it can be supported by other facts. As an example, “four times two is equal to six” is a false proposition that might be given by a preschooler. Because the statement does not demonstrate an accurate multiplication operation, it is in contradiction with the facts about multiplication.

For this research, I defined knowledge as cognitive products, which consist of procedural and conceptual propositions that might be projected onto facts negotiated by others as valid. Any statement in collected data was labeled as knowledge as long as it was used with certainty, and was linked to facts that might be the consensus of a group of professional people in the domain; otherwise it was identified as a belief.

Mathematical Knowledge for Teaching

Regarding that definition of knowledge and its difference from beliefs, researchers in the field of education investigated the scope of knowledge for the teaching profession (Nespor & Barylske, 1991; Clandinin & Connely, 1996). In the second half of the 1980s, scholars designed several new teacher knowledge models (Leinhardt & Smith, 1985; Ogle, 1986; Shulman, 1986; Freeman, 1989). Generally speaking, researchers first focused on how to differentiate subject-matter knowledge from pedagogical knowledge, until the introduction of *Pedagogical Content Knowledge* (PCK) by Schulman in 1986. Shulman (1986) formed a teacher knowledge model comprised of three kinds of knowledge: 1) content knowledge, 2) pedagogical content knowledge (*PCK)*, and 3) curricular knowledge. In his model, he described content knowledge (or subject-matter knowledge) composed of facts, concepts, and understanding of structures within a given subject. He described *PCK* as a new form of subject-matter knowledge utilized during instruction to facilitate students' comprehension. Several educational researchers in and outside of mathematics education looked at PCK within the classroom context and created new teacher knowledge models (Carpenter, Fennema, Peterson, & Carey, 1988; Howey & Grossman, 1989; Grossman, 1990; Even, 1993).

Ball and Bass (2000) pointed out there is a need to connect content and pedagogy specifically for teaching mathematics. Researchers in the field of mathematics education (Ball, Lubienski, & Mewborn, 2001; Hiebert, Gallimore, & Stigler, 2002) criticized the focus on the examination of mathematics curricula to identify mathematics knowledge needed for teaching. They claimed that classrooms should be the research sites in order to reveal the type of professional knowledge needed by mathematics teachers. These criticisms, along with the shift of emphasis from formal to practical knowledge in the field of education, led Ball and her colleagues (Ball, Lubienski, & Mewborn, 2001; Ball, 2003; Ball, Thames, & Phelps, 2008) to develop and introduce *Mathematical Knowledge for Teaching* (MKT).

In the MKT framework mathematics knowledge for teaching was first divided into subject-matter knowledge and PCK. These two sides were then further partitioned into three knowledge components (see Figure 6.1) (Ball, Thames, & Phelps, 2008). Because this study focuses on SCK development, I next unpack SCK as a construct and how it differs from CCK.

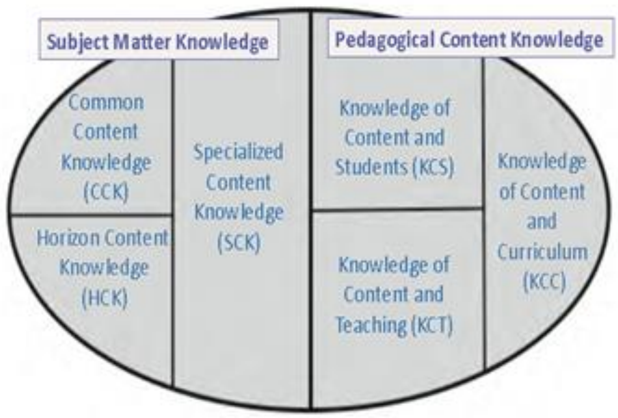


Figure 6.1: Representation of the MKT Framework (Ball, Thames, Phelps, 2008)

Common Content Knowledge (CCK): Hill, Sleep, Lewis and Ball (2007) defined CCK as "knowledge of mathematics that was common across professions and available in the public domain" (p. 131). With the availability of CCK, teachers would be able to "compute, make correct mathematical statements … , solve problems" (Hill, Schilling, & Ball, 2004, p. 16), "know the material they teach … , recognize when their students' give wrong answer or when the textbook gives an inaccurate definition …, use terms and notations correctly" (Ball, Thames, & Phelps, 2008, p. 399). The authors underlined that CCK was common to any profession that uses or applies mathematics.

Regarding these definitions, I defined CCK for this study as mathematical knowledge, which an undergraduate student, who was not necessarily majoring in mathematics education, might develop through his/her university study. CCK is factual, conceptual, procedural and algorithmic knowledge that enables person to recognize mathematical facts and procedures, define concepts correctly by using mathematically accurate terms and notations, and distinguish the correct answer from the incorrect ones for a given problem.

Specialized Content Knowledge (SCK): SCK is the mathematical knowledge exclusively necessary for the teaching profession (Ball, Hill, & Bass, 2005). The following list of teacher skills would describe what constitutes SCK and how it is different from CCK:

- Representing mathematical terms and operations visually,
- Using common procedures with reason,
- Understanding mathematics behind students' unusual procedures and generalizing them if needed,
- Constructing real life problems related to mathematical concepts,
- Examining and unraveling the mathematical source of students' errors (Hill, Schilling, & Ball, 2004; Ball, Hill, & Bass, 2005; Hill et al., 2007; Ball, Thames, & Phelps, 2008)

Different from CCK, SCK is knowledge used for contributing to students' learning, but not taught directly to students. That said, for a mathematics teacher, it is necessary to acquire and develop SCK, a special kind of subject-matter knowledge, not for teaching to students, but for utilizing it when needed. Conceptual knowledge can be part of teachers' SCK as long as the teacher did not aim or plan to teach it to students.

Literature addressing SCK

As I mentioned in the introduction of this paper, there are a limited amount of studies examining pre-service teachers' SCK development. Morris, Hiebert and Spitzer (2009) investigated how pre-service mathematics teachers develop their SCK. The authors in this study defined SCK as necessary mathematical knowledge for teachers to develop skill in specifying and unpacking learning goals into sub-concepts. Their study showed that pre-service teachers could manage to identify sub-concepts for a learning goal in supportive contexts such as solving a given problem by themselves or examining students' mathematical errors. However, teachers could not use their SCK for planning, evaluation, teaching and learning of these concepts.

Bair and Rich (2011) examined the same phenomenon over the span of two mathematics content courses. Some teachers were better in unpacking their SCK while teaching than others. This exploratory study demonstrated that teachers organized their classes by using their PCK and SCK when their SCK was higher. Pre-service teachers with lower SCK only made trivial changes with numbers when they were asked to create follow-up problems regarding the same learning goal. When they were asked to create similar problems with non-trivial numerical changes, these teachers could not keep the difficulty level of the problem the same. In addition, the study indicated CCK is not always prerequisite knowledge for higher SCK development.

The literature examining the development of SCK through the use of technology was also limited. Silverman and his colleagues (Silverman & Clay, 2009; Silverman, 2012) studied the effect of an online collaborative environment on pre-service and in-service K-12 teachers' MKT in general within an online geometry and algebra content course. In the online collaboration environment, teachers privately posted what they thought and which ways they used when they solve problems. After teachers submitted their reasoning for their solutions, the instructor of the course led an online whole-class discussion around teachers' reasoning, solutions and the instructional objectives. The study revealed the online discussions helped teachers gain pedagogical insights for their teaching practices. Unfortunately, this study did not address how or which type of technology can trigger or limit this development. Furthermore, authors emphasized how online collaboration impacted teachers' PCK rather than their SCK under the MKT framework.

Defining Belief as a Construct

In order to actualize reform ideas in practice, Pajares (1992) claimed teachers' beliefs should be the focus of educational research. In his literature review, Pajares (1992) defined beliefs as dispositions towards actions and behaviors as a result of previous experiences, and representations of reality with enough valid and true propositions (Pajares, 1992).

The difference between knowledge and beliefs is not very straightforward. According to Pajares (1992), beliefs are more static and knowledge is more dynamic in terms of their capacity to change over time. Beliefs are mostly indisputable truths for believers, which would be held according to personal interpretations of experiences.

Kagan (1992) differentiated beliefs from knowledge by linking the former to opinions and the latter to facts. The author defined teacher belief as a "provocative form of personal knowledge that is generally defined as pre- or in-service teachers' implicit assumptions about students, learning, classrooms, and the subject matter to be taught" (p. 65). Regarding her definition, knowledge and beliefs are intertwined constructs.

The definitions given by Pajares (1992) and Kagan (1992) guided the operational definition of beliefs for this study. Beliefs were defined as cognitive entities, which are formed and emerged from individuals' experiences, and interpretations of the happenings around them. I agreed that knowledge was a related cognitive construct to beliefs, but I viewed and operationalized them separately for this study. To differentiate beliefs from knowledge, I examined the degree to which teachers' statements or observed behaviors indicated opinions, which are more individualistic, rather than facts, where their correctness can be negotiated by a group of experts in the field (Kagan, 1992).

Research looking for the effect of beliefs on teachers' content knowledge is limited. Hollingsworth' study (1989) was one of these studies that examined the link between teacher beliefs and their content knowledge development. In this study, the author traced teachers' preprogram beliefs and how they shaped their knowledge development and changed the trajectory for their beliefs. Participating teachers in this study interpreted the graduate program and courses according to their preprogram beliefs, which also formed their teaching practices later. The study showed that having viewpoints contrary to the program goals helped teachers reflect on teacher educators' reform-based instructional activities. In other words, different viewpoints created different PCK for them to use in their future classes. Unfortunately, the difference between teacher beliefs and knowledge was not clear in this study. In addition, Hollingsworth (1989) only highlighted how teachers' beliefs affected their PCK development, but not their subject-matter knowledge development.

Belief Categorization for Mathematics, Teaching and Technology

In order to investigate their interaction with teachers' content knowledge, teachers' beliefs were examined in three dimensions: 1) beliefs about mathematics, 2) beliefs about teaching, and 3) beliefs about technology. There have been several frameworks and labels used to identify mathematics teachers' views and conceptions of mathematics (Skemp, 1978; Ernest, 1989; Lerman, 1983; cited in Thomson, 1992), beliefs about teaching (Kuhs and Ball, 1986; cited in Thompson, 1992; Grant, Hiebert & Wearne, 1998), and views about technology (Groth, Spickler, Bergner and Bardzell, 2009; Chen, 2011). In this study, I employed Ernest's categorization (1989) for beliefs about mathematics, Kuhs and Ball's framework (1986; cited in Thompson, 1992) for beliefs about teaching, and Chen's categorization (2011) for beliefs about technology.

Ernest (1989) defined the nature of mathematics in three ways: 1) Problem solving view of mathematics, 2) Platonist view of mathematics, 3) Instrumental view of mathematics. In the problem solving view of mathematics, people view mathematics as a human construction through exploration of mathematical problems. According to the Platonist view, mathematics can be discovered as a static entity that consists of concepts and their relationships.

Kuhs and Ball (1986; cited in Thompson, 1992) created a categorization of orientations for teaching and learning mathematics in four viewpoints: 1) learner-focused, 2) content-focused with an emphasis on conceptual understanding, 3) content-focused with an emphasis on performance, and 4) classroom-focused. In the *learner-focused view*, teachers focused on students' construction of their own mathematical knowledge with the support of teacher in the classroom. In the *content-focused view with an emphasis on conceptual understanding*, teachers would prioritize mathematical content, concepts, their relationships, and its comprehension with connections to the big picture. On the other hand, students' acquisition of rules, procedural skills, problem-solving techniques with efficiency, the use of certain mathematical terms, and their automation would be aimed by the teacher in the *content-focused view with an emphasis on performance*. Finally, teachers with a *classroom-focused view* would focus on imitating effective teacher behaviors described within the process-product studies, such as managing classroom effectively, pursuing higher expectations from students, transmitting clear structure for the content, use appropriate assessment techniques and feedback.

Chen (2011) proposed two types of teacher beliefs about technology: 1) instrumental and 2) substantive beliefs. Teachers having instrumental beliefs about technology would consider technological devices as tools to improve the efficiency of their instruction without considering whether there is any modified influence on students' cognitive processes or learning. On the other hand, teachers who hold substantive beliefs perceive technology as an aid for students' learning and understanding. With substantive beliefs, teachers create a new medium in which learners and technological devices engage in reciprocal interactions, resulting in stronger student understanding of mathematical concepts.

Methods

Context

A graduate-level geometry course, Geometry for the Middle Grades, which took place at a Southeastern research university in the fall semester of 2013, served as the research site for this study. 16 PSTs who sought a Master's of Arts in Teaching enrolled in the course. Individuals who have received to have a bachelor's degree in another field choose The Master of Arts in Teaching (MAT) in middle level education graduate degree program to transition to a teaching career in middle level education through initial certification. In the program, PSTs take a 4-course mathematics content sequence consisting of Number & Operations, Algebra, Geometry, and Probability & Statistics. The Geometry for the Middle Grades course was selected as the research site for this study because PSTs enrolled in this course were expected to use the dynamic geometric software, The Geometer's Sketchpad (GSP) (Jackiw, 1995), flexibly and fluidly with the course's tasks.

The learning goals for the Geometry for the Middle Grades Course included: describing typical middle grades geometry content; exploring course standards for middle grades geometry; using dynamic geometric software flexibly and fluidly to solve problems in geometry; getting experience with manipulatives during problem-solving tasks and recognizing their use in geometry classrooms and explaining, justifying, and writing proofs related to course content (Course Syllabus, 2013). During the first day of the course, students downloaded GSP onto their personal computers. Each class meeting took three hours weekly for a total of 13 weeks during the semester.

In her teaching philosophy, the instructor emphasized the importance of inquiry-based teaching and learning, reflection, metacognition, assessment of PSTs' previous knowledge and their motivation for teaching and learning mathematics. The majority of her classroom activities were open to PSTs' different interpretations, analysis and conduct. Her typical instruction started with the presentation of an open-ended problem and a small whole-class discussion to engage PSTs. After that, she allowed PSTs to form a group to work on the presented problem collaboratively and to explore the mathematics within the problem. The final part of a typical instruction included an explanation phase by which PSTs presented their methods for the problem and discussed with others. The instructor deliberately chose these groups whose solutions involved errors in order to allow others to be aware of possible misconceptions their students might encounter in the future. The instructor also used exit tickets at the end of almost each class meeting to help PSTs reflect on the mathematical focus of the class meeting, its connection to their previous knowledge, and its relationship to the real world. The instructor was also reflective on her teaching methods. During one of the class meetings, she asked PSTs to evaluate her instruction in terms of content, assessment techniques, and teaching methods.

Participants

PSTs participating in this study had varied educational backgrounds, having previously earned bachelor degrees in areas such as: psychology, religion, business administration, economics, marketing, financial management, communication, electrical and computer engineering, nursing, physical sciences or engineering. All 16 PSTs enrolled in the course volunteered to participate in the study.

The instructional technology experience or knowledge of the PSTs participating in this study was limited at the beginning of the semester. They also had not taken any geometry courses or reviewed geometry content since their experiences in high school. Only one of the 16 PSTs had any teaching experience prior to the beginning of the course. This teacher taught physical education, science and computer science for two years, but he still did not have any experience with instructional technologies for mathematics or geometry.

Out of 16 PSTs, six focal participants were selected to participate in the semi-structured interviews. For the selection of these six focal participants, an entrance survey in order to identify initial beliefs about mathematics, teaching and technology was administered to all 16 PSTs at the beginning of the semester. Three PSTs were excluded from the selection of focal participants due to their unclear responses in the survey. The rest of the PSTs' responses to the belief-related question about mathematics were categorized into two (*instrumental* versus *platonic* view of mathematics) regarding Ernest's framework (1989). None of the participants' responses for this question show evidence of view of mathematics emphasizing mathematics as a human construction and invention. For these 13 PSTs, I secondly used Kuhs and Ball's framework (1986, cited in Thompson, 1992) and categorized their responses to the belief-related question about teaching into three (*learner-focused, content-focused with an emphasis on conceptual understanding* and *classroom-focused*). No PSTs' responses demonstrated evidence of teaching belief that could be categorized as content-focused with an emphasis on performance. Finally, I categorized these PSTs' responses to the belief-related question about technology. I used Chen's framework (2011) and labeled their responses as *instrumental* and/or *substantive*. Some PSTs' responses showed evidence for both of the categories, which were labeled as *both*. Table 6.1 presents findings from this preliminary analysis for belief categorization:

Participants[3]	Math Beliefs	Teaching Beliefs	Technology Beliefs
Kristin	Instrumental	Classroom-focused	Substantive
Derek	Instrumental	Classroom-focused	Both
Cameron	Instrumental	Content-focused emphasizing understanding	Both
Victor	Instrumental	Content-focused emphasizing understanding	Instrumental
Kathleen	Instrumental	Learner-focused	Instrumental
Leonard	Instrumental	Learner-focused	Substantive
Richard	Platonic	Classroom-focused	Instrumental
Katherine	Platonic	Classroom-focused	Instrumental
Cindy	Platonic	Classroom-focused	Both
Karl	Platonic	Content-focused emphasizing understanding	Substantive
Abby	Platonic	Content-focused emphasizing understanding	Both
Erica	Platonic	Learner-focused	Both
Samuel	Platonic	Learner-focused	Substantive

Table 6.1: Participants' Preliminary Belief Profiles at the Entrance Survey

Accounting for variation in responses to belief-related questions in the entrance survey (Table 6.1), I selected six focal participants (Kristin, Cameron, Kathleen, Richard, Karl and Erica). For this paper, I present the results for five of them. I elected not to include Erica due to the dearth of richness in her data. Table 6.2 demonstrates each of these five participants and their previous experiences:

Focal Participants	Age	Previous Professional Background
Kristin	53	Nurse
Cameron	28	Business, teaching science and computer science
Kathleen	35	Hair stylist, management, interior design
Richard	43	Business, pharmacy
Karl	21	Economics

Table 6.2: Focal Participants' Previous Experiences

Data Collection

This paper addresses one part of a larger study. For this facet of the research, I collected and examined two sources of data: 1) surveys to measure beliefs; and 2) participant interviews (Yin, 2008). The collection of different types of data, such as

[3] Participants' names are pseudonyms.

surveys and interviews allowed me to triangulate the data through multiple sources of evidence. This also created a chain of evidence and helped establish validity for my findings.

A survey was administered at the beginning of the study. During the first class meeting of the geometry course, 16 PSTs completed the *Entrance Survey,* which was designed to collect information about their background and to establish their current beliefs about mathematics, teaching, and technology. This survey was originally adapted from a survey designed by Schmidt and his colleagues (2009) to assess PSTs' TPACK. Reliability for each knowledge component was greater than 80% (Schmidt et al., 2009). Because beliefs about mathematics, teaching and technology were examined for this study, open-ended items were added to the end of the survey for that purpose. Preliminary findings from this survey about PSTs' beliefs and previous experiences with technology were followed to identify six focal participants for interviews.

A second data source was a series of three semi-structured interviews. Semi-structured interviews included 1) conversations around a geometry task that participants were asked to solve utilizing GSP, and 2) belief-related questions. The geometry tasks (Table 6.3) embedded within the GSP were designed to unravel participants' SCK. The tasks were designed to provide insight into participants' geometric thinking both with and without GSP, as well as participants' use and development of SCK. These interviews were administered during the second week of September, October and November. Interviews were audio and video recorded to capture discussion as well as to document participants' actions as they engaged in the geometry tasks.

Interviews	Date	Tasks	Tasks' Description
1st Interviews	16-20 September 2013	Square Construction	Participants were guided to construct a square by using a circle and parallel and perpendicular lines
2nd Interviews	21-25 October 2013	Triangle Inequality Theory	Participants were guided to construct the Triangle Inequality Theory by interacting with GSP
3rd Interviews	18-22 November 2013	Inscribed Circle of a Triangle	Participants watched animations in order to develop the relationship among the area, perimeter of a triangle and the radius of the circle inscribed in the triangle.

Table 6.3: Description of Geometry Tasks during Interviews

The rest of the first interview included belief-related questions, which were in two forms. At first, the interviewer asked open-ended questions about participants' beliefs about mathematics, teaching and instructional technology. After that, the interviewer asked more guided questions during which participants were asked to read statements, to choose if they agreed, and to rank the ones they chose. These statements were created with respect to the theoretical frameworks elaborated in the previous section for each type of belief separately. For example, there were separate statements representing instrumentalist, Platonist and problem solving view of mathematics; the choice of these statements and rankings by participants demonstrated their inclination towards a belief about mathematics in Ernest's framework (1989). Questions related to beliefs about mathematics, teaching, and technology were identical in the first, second, and third interviews. The same questions addressing beliefs were included in all interviews to increase the ability to form a belief categorization for each participant coherently, and to see how their beliefs might differ throughout the semester. In other words, posing the same questions three times triangulated the data, and strengthened evidence for each participant's beliefs about mathematics, teaching and technology.

Methodology and Data Analysis

A holistic multiple case study design was used as the methodology for this paper (Yin, 2008), where five PSTs having different professional background became the units of analysis. This study's main purpose is to understand the phenomenon of knowledge development for teachers and how their beliefs about the profession as well as the discipline and instructional technologies can be linked into this phenomenon. In this respect, the case study approach was the most appropriate methodology to examine factors and their relationships of real-life phenomenon, within a specific context.

Data analysis started following the administration of the entrance survey at the beginning of the semester. The main purpose of the data analysis at that time, as mentioned, was to select participants for interviews throughout the semester. To map their responses in the entrance survey onto the theoretical frameworks for beliefs about mathematics, teaching and technology, pre-determined descriptive codes (Table 6.4) were used. The same framework was used to categorize PSTs' beliefs for each interview separately.

Belief Type	Codes	Description
Math	Instrumental	While defining mathematics, the participant refers to algorithms, facts and laws as if there is no relationship among them and/or to the physical world.
	Platonic	The participant considers mathematics is common around the world, objective with no exception, exact, and having one correct answer to a given mathematical problem. While defining mathematics, the participant does not think that it is a bunch of rules. The participant defines mathematics as consisting of concepts that are related/connected one another and to the real life decisions.
	Problem Solving	The participant is aware of the fact that mathematics is dependent on society, civilization, and social experiences that people go through. The participant defines mathematics as a man-made construct that can be communicated.
Teaching	Learner-focused	The participant prioritizes students' learning, understanding, abilities and interests. The participant exemplifies how to achieve students construct their knowledge and come to know concepts by themselves.
	Content-focused emphasizing understanding	While describing how to teach mathematics, the participant refers to the content and conceptual teaching. The participant considers teacher should clarify why s/he uses a specific strategy while delivering the content.
	Content-focused emphasizing performance	The participant prioritizes students' preparation to the tests and their achievement in these tests for their teaching.
	Classroom-focused	The participant describes teaching as to deliver their knowledge or give rules required. While describing teaching mathematics, the participant refers to assessment, instructional preparation and classroom management techniques.
Technology	Instrumental	The participant considers technology is necessary for efficiency. It allows to save time during instruction and to deal with managerial tasks such as grading. Technology is advantageous to demonstrate the content. It is a new way for tutoring through videos. The participant considers technology might be disadvantageous because it is a tool giving the correct answer.
	Substantive	Technology does not only help teacher for instructional efficiency, but also enhances students' mathematics ability, and aids students' learning, reasoning, sense making, and conceptual understanding. The participant considers that technology should not be behaved as another means for old technologies such as the blackboard. In addition to its efficiency, the participant views technology to be used in order to enhance the math ability.

Table 6.4: Pre-determined Codes for Beliefs about Mathematics, Teaching and Technology

All interviews were transcribed, and the geometry tasks during the interviews completed with GSP were narrated and elaborated through video analysis to examine

PSTs' SCK. For each geometry task, a rubric to assess PSTs' SCK was constructed. PSTs' errors or misconceptions uncovered during the task were explored. The exploration of these errors also enabled me to decide if a PST demonstrated the identified SCK for the task fully or partially.

PSTs also were asked to select statements to represent their beliefs about mathematics, teaching and technology (see Appendix). These statements were also used to triangulate my finding of belief categorization for each interview and for each PST. Responses to belief-related questions during interviews were again coded with respect to the pre-determined codes and their definitions in Table 6.4. As a result, each PST was identified with *one* type of belief about mathematics, teaching and technology for each interview. For example, the following text is an excerpt from the third interview with Kristin when she was asked how mathematics should be taught in class.

> Interviewer: OK. What about your views about teaching? How should mathematics be taught?
>
> Kristin: The first thing you should think about is your students and their abilities as math learners. Because by the time they get to the middle school, a lot of them had an experience that they either had a lack of confidence in math abilities, or they're confident in their math abilities…And you have to assess what the students think about their math abilities. That would give you inside into maybe their best learning styles.

This text was coded as a *learner-focused teaching* belief because she referred to students' mathematics abilities, who would fit into the definition of that type of belief in Table 6.4. In addition, from the given statements, she selected "learners' background and interest is important to begin with in my teaching". These data allowed me to categorize Kristin's belief about teaching during the second interview as Learner-focused teaching.

After I analyzed PSTs' SCK and beliefs for each interview, I created a profile for each PST nested within each interview in order to examine the relationship between their SCK and beliefs. These profiles within each interview were compared to one another to answer the research questions for this paper.

Results

In this section, I present results for each interview including PSTs' identified SCK for the specific geometry task and their beliefs. After the presentation of the overall picture for each PST, those PSTs for whom their SCK *might* be linked into a specific type of belief about mathematics, teaching or technology were described and discussed further.

Interview 1

The first interview included a construction of a square task. The actual task is given below:

> **Square Construction**
> Susan claims she can construct a square by using the properties of a circle and lines having parallel or orthogonal properties. She began by constructing a circle by its center and around a point, and then constructing the radius (*see* Figure 6.2). Can you complete the construction? What do you think the student would do next? Predict their method based on what is described.

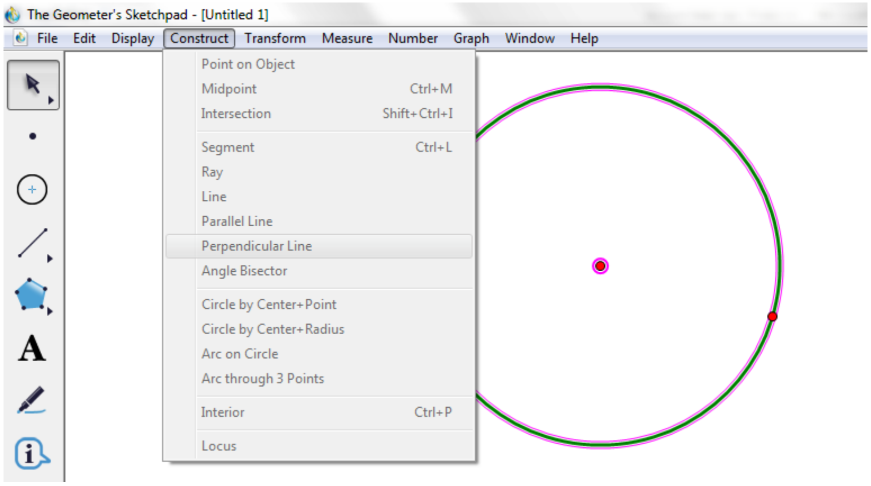


Figure 6.2: Screenshot from GSP Square Construction Task

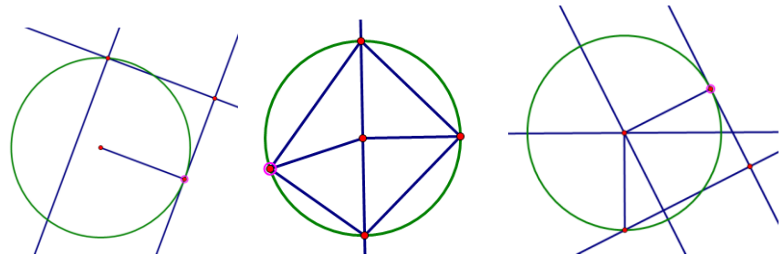
The SCK associated with this task was defined as the PSTs' ability to predict and/or complete a student's procedure to construct a square with GSP according to a given scenario. In this case, PSTs were told that the student started to construct a square by beginning with a circle and its radius. From this opening step, there are three possible ways to arrive at the construction a square, each of which produces a different sized square. These are: 1) a square constructed from the radius of the given circle; 2) a square circumscribed around the circle, or 3) a square inscribed in the circle.

Of these three procedures to demonstrate SCK, Karl was the only PST to construct a square circumscribed around the circle (Table 6.5); Kathleen was the only PST who constructed a square inscribed in the circle. The other three PSTs constructed a square from the radius of the circle. As technology was utilized during the completion of the task, technological expertise and knowledge was also a part of the expected SCK. PSTs' lack of technological expertise with the program led me to assess their SCK as partially demonstrated. During the first interview, 2 PSTs (Cameron and Karl) fully demonstrated the identified SCK. These participants' procedure prediction and square construction were complete and correct, without containing any technological or mathematical errors. The other three PSTs partially demonstrated the SCK expected for the task (Table 6.5), in that they understood a path to take, but made some sort of mathematical or technological error in their square construction.

Participants	Type of Square Constructed	Construction Error Included (Yes/No)	Root of the Error	SCK Demonstrated (Fully/Partially)
Cameron	From the radius given	No	-	Fully
Karl	Circumscribed around the circle	No	-	Fully
Kathleen	Inscribed in the circle	Yes	*Technological*: Approximation of location of vertices	Partially
Kristin	From the radius given	Yes	*Technological*: Approximation of location of vertices	Partially
Richard	From the radius given	Yes	*Technological*: Approximation of location of vertices	Partially

Table 6.5: Emerging Codes for PSTs' Square Constructions with GSP

Kristin, Kathleen and Richard's constructions included technological errors (Figure 6.3a, 6.3b, and 6.3c respectively). Even though each of them was aware of the geometrical properties of a square, the PSTs did not fully incorporate their technological knowledge during their constructions, instead relying on "eyeballing" and approximations to locate vertices of the square on GSP. As a result, the squares they constructed did not preserve the required properties when manipulated within the dynamic software.



a b c

Figure 6.3: Three PSTs' Errors during the Square Construction

PSTs' responses to the belief-related questions were coded as in Table 6.6. Out of five, only Kathleen showed a different belief about mathematics than what she reported in the first interview. Her statement during the first interview indicated problem-solving type beliefs about mathematics that emphasized mathematics as a human construct dependent on society, time and civilizations:

> …we're exposed to totally different things than people that first found these things [for mathematics], the kind of civilization and society they were living in. So we can rediscover it like a first time experience, but we're probably not going to rediscover it in the same social, emotional, scientifically type way. (Kathleen, first interview)

Participants	Belief about Mathematics	Beliefs about Teaching	Beliefs about Technology
Cameron	Platonic	Classroom-focused	Substantive
Karl	Platonic	Content-focused emphasizing understanding	Substantive
Kathleen	Problem Solving	Classroom-focused	Instrumental
Kristin	Platonic	Classroom-focused	Instrumental
Richard	Platonic	Classroom-focused	Instrumental

Table 6.6: PSTs' Beliefs during the First Interview

Table 6.5 and 6.6 together indicate that the PSTs whose square construction included a technological error (Kathleen, Kristin, and Richard), also held instrumental beliefs about technology. For example, during the first interview, when Kathleen was asked for her beliefs about technology, she stated that “[she] likes [technology] as far as being able to be used after hours for like tutoring because a lot of those things can be done inexpensively”. This statement was coded as an instrumental belief about technology because it did not include any reference to the benefit of technology for learning concepts, but rather as a means to find other means to study at home and to be prepared for the class. Until the first interview, Kathleen and other PSTs had experience with GSP only for one month. This brief experience with technology did not allow Kathleen to view GSP as a technology to learn with.

Kathleen had a broader belief about mathematics that allowed her to view it as a human construct. She constructed the square in a different way. However, her construction was not error-free. Her construction experience with GSP might have been another reason for her to view technology not as a learning partner yet. In other word, her lack of full SCK demonstration while using GSP during the square construction task might have been another reason for her instrumental belief about technology.

Richard and Kristin viewed technology in a similar way to Kathleen during the first interview. Kristin viewed technology as tools to rely on for measurement precision and ability to demonstrate things in an easier manner. Even though he did not mention these kinds of advantages of technology, Richard also demonstrated an instrumental belief about technology. Even though he stated that the priority to use technology should be for learning, his interview did not indicate any consideration to integrate it as a mind extension. During the first interview, Richard and Kristin’s square construction also included technological errors (see Table 6.5), which resulted in a partial demonstration of SCK for the task.

On the other hand, Cameron and Karl represented substantive beliefs about technology. For example, Cameron described instructional technology as an aid for conceptual understanding:

> …Even if you don't fully understand a concept in geometry for example, you can make sense of it. As long as you know how to work with the program, it may help you to understand geometry a little better. It makes more sense to them when they see the circles are connected. The radius of two circles, in relation to… you know two circles meet make a triangle. Or how to draw a perpendicular line. I think they would help them to better understand it visually. (Cameron, first interview)

A substantive belief about technology for Karl and Cameron might have enabled them to fully demonstrate SCK for the square construction task. Whereas, Kristin, Kathleen and Richard's instrumental belief about technology might have been the reason for them to have technological errors during working with the same task.

Interview 2

The geometry task during the second interview asked PSTs to hypothesize the reason for a student's error while s/he was working with GSP. The actual task is below:

> **Triangle Inequality Theory**
>
> You taught that, in any triangle, one side has to be smaller than or equal to the sum of two other sides, and larger than or equal to the absolute value of the difference between the other two sides ($|a-b| \leq c \leq (a+b)$). A student using GSP states that a triangle having sides measured 2, 4, 5 inches cannot be formed (*see* Figure 6.4). However, regarding the triangle inequality, a triangle should be formed with these combinations. What should be the reason for the student's error?"

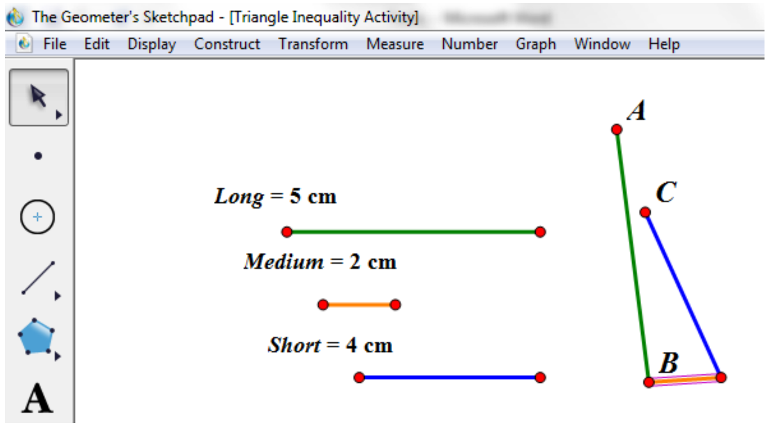


Figure 6.4: Screenshot from GSP Triangle Inequality Task

To predict the reason behind the student's error, PSTs interacted with the applet, dragged vertices of the triangle and played with sliders to change side lengths of the triangle. In my analysis of the situation, the student's misconception could be explained by three possible reasons, the student: 1) lacks obtuse triangle conception and spatial reasoning (*mathematical reason*); 2) focused on a special type of triangle construction (*mathematical reason);* or 3) lacks knowledge about the program's affordances (*technological reason*).

PSTs did not need to use the technological affordances of GSP in the same manner as the task in the first interview. As such, PSTs' SCK was compared according to the number of reasons they predicted for the student's error. Table 6.7 demonstrates how PSTs speculated on the given scenario and what type of reason they gave for the student error.

Participants	Identified Error Type	SCK Demonstrated (Fully/Partially)
Cameron	*Technological*: Lack of Knowledge about Program's Affordances	Partially
Karl	*Mathematical*: Focusing on a Special Type of Triangle Construction *Technological*: Lack of Knowledge about Program's Affordances	Partially
Kathleen	*Mathematical*: Lack of Obtuse Triangle Conception and Spatial Reasoning *Mathematical*: Focusing on a Special Type of Triangle Construction *Technological*: Lack of Knowledge about Program's Affordances	Fully
Kristin	*Mathematical*: Lack of Obtuse Triangle Conception and Spatial Reasoning *Technological*: Lack of Knowledge about Program's Affordances	Partially
Richard	*Technological*: Lack of Knowledge about Program's Affordances	Partially

Table 6.7: Emerging codes for PSTs' SCK during the Second Interview

Each of the five PSTs considered the error was mainly the result of lack of competency with the program. Even though the applet would allow users to drag two different points, PSTs surmised the student only dragged one point, and as a result, could not construct a triangle with the given side lengths (*see* Figure 6.4).

Cameron categorized the student's error more of a technological error. While explaining his reason, he considered that the student did not try to move more than one side on GSP:

Interviewer:	…What might be the mathematics behind this student's error? What do you think?
Cameron:	[He brings two open points of the triangle together]. So, I imagine that the student might have the error by moving one side or the other. For example, if this length [showing the shortest side] was here, then the student would only try to move this [the point C] and say I cannot because… that side is too short. Or they are trying to do this way [point A]. They are like, OK, we aren't touching the points. They were not trying to form an angle part where these two sides were really met to actually form the other side of the triangle.
Interviewer:	Do you think it is a kind of math problem or a technology problem?

Cameron: More of a technology.

As Cameron, Richard did not refer the error onto the properties of the triangles or its angles, but viewed it as a more of technical error related to the movement of sides, angles and the use of one vertex of the triangle:

Interviewer: ... What should be the reason for the student's error? Or what is the math behind the student error? My first question is what the student error is here.

Richard: She is only moving two sides. She has three sides. First thing I would give her "look at segment bc and bring it closer to segment ba". Because this triangle is going to be much flatter… They weren't looking... They weren't moving angle C. Because they had left C. when you originally approach with this cause C was here. So the error was they only use this as only pivot and they didn't move C.

Interviewer: OK. Would you say this is a mathematical error or a technological error?

Richard: To me, it would be a more technical error… I would say the student is making more a technical error.

Both Cameron and Richard's explanation during the second interview were coded with technological reason which might be attributed to lack of technological knowledge about the program's affordances. In addition to technological reason, Kristin's interview showed evidence for a mathematical reason as well. She first questioned students' knowledge of GSP while doing this error. Kristin also stated that the student might have had the error because of his/her lack of spatial reasoning:

Interviewer: ... Can you predict like what kind of error is she doing by setting that? […] So what is the mathematics behind it?

Kristin: Is the student familiar with GSP?

Interviewer: Yeah.

Kristin: I think the way it was given, it looked like you cannot manipulate it so that the space would be closed. I am thinking of experiences I had with people who are challenged with seeing things that are not there yet, and representing, and manipulating things.

Interviewer: Do you think this is a kind of technology related problem or math related problem?

Kristin:	I think it is spatial reasoning problem. I think some people are challenged to project something that they do not see, cannot touch or manipulate themselves.

The same as Kristin, Karl thought the student's action in the scenario included both technological and mathematical error. His first statement below was coded as the technological reason. In addition to that, he stated that the student might have been trying to construct a special type of triangle such as right triangle. The second statement was coded as mathematical reason that might be linked to the student's focus on a special type of triangle construction:

Interviewer:	What do you think the error is?
Karl:	I would say the error is that they didn't realize they could move *A* and *C*. Because what I would do, can I do it…you could move that one. You can line them up that way. So I think he just has… he doesn't realize that he can move...
Interviewer:	So what do you think the problem, the math is behind that problem, this error? What kind of mathematics? Is there any mathematical problem or anything else?
Karl:	Both. Maybe he's trying to make a right triangle or an equilateral triangle, which he might be trying to make some sort of perfect triangle that won't be made with these measurements.

Kathleen identified all three reasons for the student's error. During the interview, she implied that the student needed to make an obtuse angle in order to have a triangle with given side lengths. Secondly, Kathleen labeled the student's error as a technological error as much as a mathematical error. Her statement below about the movement of the sides of the triangle and merging two points on GSP was coded as a technological error:

Kathleen:	They didn't move point C out far enough… [the student] didn't make angle A obtuse enough…
Interviewer:	And do you think the error is math related or technology related?
Kathleen:	I think it might be both. They wouldn't have the mathematical understanding and they were moving one, possibly moving one side and not moving the other side or not able to picture the two coming together because of how they were moving it. Since they were just

moving one point at a time. Cause kids are more linear thinkers… Maybe they have an ideal in their mind of what a triangle has to look like. That it either has to be equilateral or maybe it has to be isosceles. And they didn't understand that you can have all different lengths.

Kathleen seemed to manage to unpack her common content knowledge by referring to special types of triangle construction as well as relating the student's error into misconception about obtuse triangles.

Participants	**Belief about Mathematics**	**Beliefs about Teaching**	**Beliefs about Technology**
Cameron	Platonic	Classroom-focused	Substantive
Karl	Platonic	Content-focused emphasizing understanding	Substantive
Kathleen	Problem Solving	Classroom-focused	Instrumental
Kristin	Platonic	Classroom-focused	Instrumental
Richard	Platonic	Classroom-focused	Instrumental

Table 6.8: PSTs' Beliefs during the Second Interview

During the second interview, four PSTs' beliefs about mathematics were coded as Platonic belief (see Table 6.8). While describing the nature of mathematics in their own words, they referred to its discovery, and emphasized the coherent relationships about concepts in mathematics. Kristin described the nature of mathematics in this way during the second interview. Her statement is representative for the responses of the other three PSTs who have Platonic beliefs about mathematics:

Interviewer: Why do you think math consists of definitions, procedures, concepts and relationships among those?

Kristin: I think until you understand the definitions of number sets and relationships and the operations, the procedures, the vocabulary, I don't think you can apply anything in math.

Interviewer: Do you think math is a discovery?

Kristin: I don't think math can be rediscovered. I think we can understand it. I think it is always there. But I don't think it may be fully understood... When I think discovery, I think finding something new. It may be a new relationship that we did not understand before.

On the other hand, Kathleen's belief about mathematics was again coded with problem solving belief (*see* Table 6.8). Kathleen described mathematics as a man-made construct that can be communicated by humans:

> …Most angles that we have and the way that we talk about them, aren't ABC. You know what I'm saying. Like that's a man-made thing. But the fact that they, the law of the angles and the similarities and stuff like that, that's not an ABC form. That's how we communicate it to one another. (Kathleen, second interview)

Kathleen maintained her instrumental beliefs toward technology however, since the second interview task did not require technological expertise, her beliefs toward technology might not have hindered her ability to reason about the student's error. Kathleen's belief about mathematics as a human construction might have led her to be more creative and consider more reasons for the student's error.

Interview 3

During the third interview, the interviewer asked PSTs to explore and determine the relationship between the area and perimeter of a triangle and the radius of its inscribed circle. PSTs were shown an animation that would suggest the relationship. The actual task for the Inscribed Circle of a Triangle is shown below:

Inscribed Circle of a Triangle

> Regarding the animation within the GSP [*see* Figure 6.5a, 6.5b, 6.5c and 6.5d for snapshots from animation], what could be the relationship? How did you understand that? Could you explain it to me?

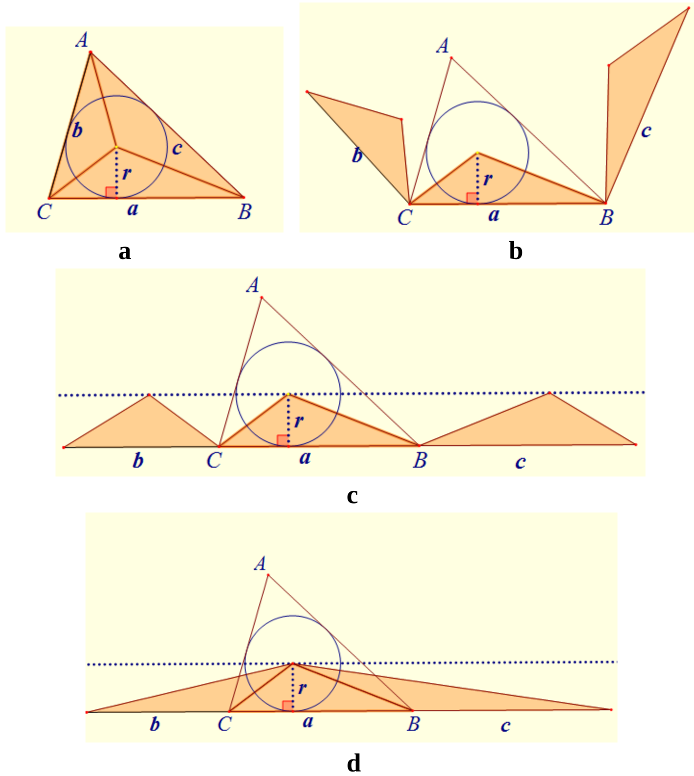


Figure 6.5: Screenshots from Inscribed Circle of a Triangle Animation

The animation (Figure 6.5a) displayed the three triangles formed by the coincidence of the three angle bisectors. Two of these three triangles were folded out (Figure 6.5b) and rotated around their vertex so that the three bases of the triangle were collinear (Figure 6.5c). Finally, the top vertices of each were translated along a line, parallel to the base and until they were coincidental with the center of the circle (Figure 6.5d).

The SCK for this activity was to understand the mathematics behind the procedures displayed in the animation and to deduce the given relationship. The animation rotated the two triangles in order to keep the area of the original triangle the same. Second, the top vertices were translated along the parallel line, maintaining the height of the triangles, and preserving the areas of internal triangles. As a result, the new triangle formed from the original one would have an equal area. Further, the new triangle would have a height equal to the radius of the inscribed circle and its base length would be equal to the perimeter of the original triangle.

Participants	**Understanding the Rotation of Internal Triangles (Yes/No)**	**Understanding the Translation of Internal Triangles' Peak Points (Yes/No)**	**Constructing the Relationship**	**SCK Demonstrated (Partially/Fully)**
Cameron	Yes	No	No	Partially
Karl	Yes	Yes	Yes	Fully
Kathleen	Yes	No	No	Partially
Kristin	Yes	Yes	Yes	Fully
Richard	Yes	No	No	Partially

Table 6.9: Emerging Codes for PSTs' SCK during the Third Interview

Each of five PSTs understood the reason behind the rotation of internal triangles (*see* Table 6.9). Richard's response to the task during the third interview represented that he understood the mathematics behind this rotation:

> When you flip them over, that is the same height each point, so they bring them back together. The height would be the same because it is the radius of the triangle. That is per se because you have three triangles. (Richard, third interview)

Cameron, Kathleen or Richard did not understand the mathematics behind the translation of peak points of internal triangles without help. On paper, the interviewer needed to show how the areas of the internal triangles would be kept the same. Because they needed further help, I coded their SCK not fully but partially demonstrated.

As an example, Cameron could not figure out why the animation operated on some movements such as translation of peak points of internal triangles after rotation. He seemed to be confused about how to find the area of an obtuse triangle and with the concept of height:

Cameron:	That was what I was using my bases when determining where the base was. You can call plane necessarily, like flat. A flat line. For example, this [EC] is a 180 degree line. So that was why I was determining that as the base, whereas this [ED] is not, because this is coming with a slope [*see* Figure 6.6].
The Researcher:	OK. If this [EC] is the base, what is the height for this base?
Cameron:	I would not think it would be point D because, again had a slope. EC as a kind of point where that comes for… I was thinking maybe you have to have a perpendicular line from line EC. You have to have a perpendicular line from that point [C]. It is where your height was… For example creating a perpendicular line from EC…
The Researcher:	…Do you want to have a perpendicular line to EC? And from which point?
Cameron:	From C t o EC.
The Researcher:	[The interviewer constructed a new perpendicular line passing through the point C and perpendicular to the line segment EC].
Cameron:	Yeah that was what I was looking for. So the point J would think would be your height.

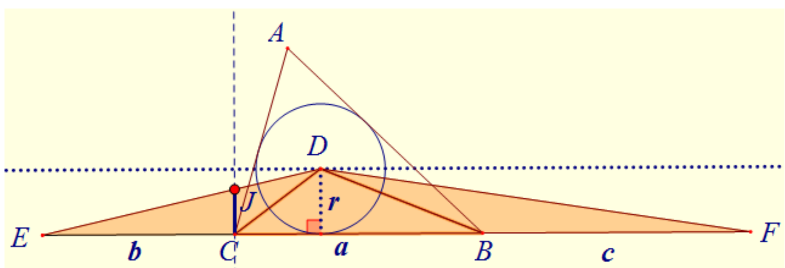


Figure 6.6: Cameron's Height Conception

After the episode above, the interviewer explained the reason behind the movement of the top points of each internal triangle. That way, he was helped to develop SCK. Since he developed SCK for this task with the interviewer's explanation, his overall interview was coded as a participant partially demonstrated SCK for this task.

Unlike the other three PSTs, Karl and Kristin understood why the peak points of the internal triangles were translated to the center of the circle, and therefore constructed the relationship with minimum help (Table 6.9). Karl recognized that the rotation allowed to keep the area of the original triangle the same. He also figured that the base of the new triangle was the perimeter of the original one.

Karl:	…they drew...segments from the angles of the triangle into the center of the circle. And they formed three smaller triangles. And then they sort of rotate it, the two upper triangles out to form a straight line, which a straight line would be the perimeter… And then they did the area of the original triangle, a, b, c, is the area of these smaller triangles added together.
Interviewer:	And when we start to move on [the peak points of internal triangles], according to you, the areas are changing?
Karl:	No, the areas aren't changing. It's just the shape of the triangle is changing…we didn't change in area because it didn't gain or lose any. We kind of just skewed the triangle back over.

For Karl, the translation of the peak points of the internal triangles after their rotation was first confusing because he thought that it changed the shape of the triangles. However, he later noticed that the animation did this in order to keep the area the same by forming a new triangle having the base measured as the perimeter of the original triangle:

> If you would have just handed that [on paper], I would have just stared at it all day. But seeing animation definitely helps because you can see...once you get, even thinking about rotating it out, I would have never thought about that…Yeah, like the rotating out. I would have never have thought to put the three triangles as one big triangle. I would have never thought to do that. So to see that helped me think that okay the perimeter is your new base, so that was really helpful. (Karl, third interview)

Kristin also recognized that the angles of the internal triangles after their rotation were changing. Her recognition allowed her to understand the mathematics behind the rotation of triangles and translation of their peak points. At first, she was uncertain about the concept of height for an obtuse triangle. By viewing the animation for a second time, she discovered this concept:

Kristin:	OK. Folded it out. [The interviewer reran the animation]. Oh yeah, the angles are changing dramatically. See that is where I am getting hung up. I am used to height as a perpendicular line. This is the highest point of that triangle. But is that considered? If I go from here to here…it would be the height.
Interviewer:	So, if you move those things, the area would be the same?
Kristin:	I believe they would be. I just think they are in different shapes… Even though these angles changed from their individual angle sides, they still represent the same area. Yeah, because you did not change anything. That is amazing.

In order to makes sense of the differences in their SCK, I examined PSTs' belief profiles during the third interview (Table 6.10). The only common belief for Karl and Kristin was the belief about mathematics, but their Platonic belief was also shared by the other PSTs.

Participants	**Belief about Mathematics**	**Beliefs about Teaching**	**Beliefs about Technology**
Cameron	Platonic	Classroom-focused	Substantive
Karl	Platonic	Content-focused emphasizing understanding	Substantive
Kathleen	Platonic	Classroom-focused	Instrumental
Kristin	Platonic	Learner-focused	Instrumental
Richard	Platonic	Classroom-focused	Instrumental

Table 6.10: PSTs' Beliefs during the Third Interview

Karl's belief about teaching was content-focused emphasizing understanding, which was not demonstrated by any other PST. He considered that content and knowledge is the most important aspect of mathematics instruction:

> I would say content comes first. Because if you have the best classroom management and you're the most likeable teacher, and you have every student's interest at heart, [but] if you don't know what you're talking about, then no learning is going to happen. Like if you don't know what you're doing, then how

can you teach or guide or facilitate students, or educate students, when you don't know what you're talking about. (Karl, third interview)

Cameron and Kathleen demonstrated partial SCK for the task during the third interview (Table 6.9). They also had instrumental beliefs about technology (Table 6.10). Kristin also had an instrumental belief about technology, but her SCK was almost in the same quality as Karl’s SCK for the task. For the Square Construction task, the data showed that participants who have instrumental beliefs about technology could not fully demonstrate SCK. However, during the Inscribed Circle of a Triangle task, Kristin as a participant having instrumental beliefs about technology managed to fully demonstrate SCK for this task.

Discussion and Conclusion

The Influence of Beliefs about Mathematics on SCK Development

This study did not show enough evidence to make any assumption about the link between belief about mathematics and SCK development with technology. Both participants who fully and partially demonstrated SCK through three interviews held the same type of beliefs about mathematics: Platonic beliefs, according to which, participants viewed mathematics as an exact and certain discipline that was embedded within the world.

Theoretically speaking, for teachers to utilize GSP to develop their SCK, both Platonic and problem solving beliefs about mathematics should be held. By viewing mathematics as a discipline consisting of concepts and relationships among them, teachers would focus on comprehending the concepts covered and grasping the relationships. Secondly, without viewing mathematics as a discipline to be constructed through investigation and discussion, it might be hard to find these relationships on dynamic geometry environment. In this study, these technologies might have helped a few participants with these types of belief about mathematics develop their SCK. For example, Kristin was uncertain about the concept of height for an obtuse triangle. Her Platonic belief about mathematics might have helped her to discover this concept while viewing the animation. However, these few instances were still not sufficient to make any claim on a relationship between this type of belief about mathematics and SCK development. There were other PSTs who held the same type of belief, but could not fully demonstrate the task's intended SCK.

Kathleen was the only PST who demonstrated a problem solving belief about mathematics throughout the semester. During the first and second interviews, she considered that mathematics might also be a human construction and invention as much as a discovery. However, she did not maintain this belief about mathematics during the third interview. One explanation for this change for her beliefs about mathematics might be her search for her personal thoughts about the nature of mathematics. Even though data for her during the first and second interviews indicated that she held a problem solving belief, these thoughts might have been ideal rather than practical. She might not have thought on this notion thoroughly, and viewed mathematics as a human construction because these ideas were reasonable for her. However, a problem solving belief might not have been evident in her way of doing mathematics in reality.

If a problem solving belief about mathematics was the dominant belief for Kathleen, the influence of the beliefs about mathematics might be explained with the quality of tasks used during each interview. The first and third interview tasks required PSTs to do mathematics. During the first interview, PSTs had to actively utilize their mathematical knowledge to construct a square using GSP. In the third interview, PSTs had to actively utilize their mathematical knowledge to comprehend, interpret, and explain the mathematics behind an animation. However, the second interview task did not require the same kind of mathematical performance or interpretation, but instead required a passive use of knowledge to hypothesize a reason for a student's error. Because of this difference, Kathleen's problem solving belief about mathematics might have helped her to consider more than one possibility for the reason behind the student's error. However, the tasks during the first and third interviews required doing mathematics and further mathematical interpretation. Platonic beliefs about mathematics might have been supported more than problem solving belief about mathematics for tasks requiring doing mathematics.

The Influence of Beliefs about Teaching on SCK Development

The same as beliefs about mathematics, the study did not show sufficient evidence to link the belief about teaching to their SCK development with technology. The dominant belief about teaching among PSTs during three interviews was classroom-focused belief where they prioritized classroom management, assessment and instructional preparation for their teaching. The reason for the dominance of classroom-focused belief about teaching might be these PSTs' lack of teaching experience. Only one

PST had a teaching experience before starting the MAT program. The rest of the participants were new to the profession of teaching. Because of this, they tended to focus on techniques on classroom management, assessment and instructional preparation. All of these skills and knowledge might have been considered as the main challenges for teaching profession.

Regarding the definitions of each type of belief about teaching, teachers with content-focused belief with an emphasis on understanding would be expected to develop their SCK in a better way with technology because of the fact that SCK is a subject-matter knowledge component necessitating deeper understanding for its acquisition. Despite this theoretical expectation, content-focused belief about teaching with an emphasis on understanding was not majorly prioritized by participants in this study during the interviews. Karl was the only participant who demonstrated content-focused belief about teaching with an emphasis on conceptual understanding. His three interviews showed the importance of content, conceptual relationships and their representations for his teaching. Karl fully demonstrated the expected SCK during the first and third interview. Compared to the other four participants, his content-focused belief for his teaching might have helped him to demonstrate better SCK profile during the first and third interviews. Still, it is difficult to make any strong claim from one participant. In addition, participants with a different type of belief about teaching (e.g. classroom-focused belief) fully developed their SCK during interviews.

The Influence of Beliefs about Technology on SCK

This study did not indicate strong evidence to make a claim for a relationship between a type of belief about technology and SCK development with technology. At the beginning of the study, I hypothesized that substantive belief about technology would be a factor in determining the benefit of GSP on SCK development. Furthermore, I also hypothesized that an instrumental belief towards technology might hinder the potential of GSP for SCK development. Cameron's and Karl's substantive beliefs about technology might have been influential for their SCK during the first interview. They did not consider GSP just as a tool independent to the learning process, but treated it as a learner partner (Jonassen, 1995). On the other hand, Kathleen, Kristin and Richard used GSP as a new medium to draw their sketches without using its affordances, which resulted in technological errors within their constructions. These teachers' lack of knowledge about how to use GSP might have been another mitigating factor for this result.

Cameron's substantive beliefs about technology did not help him to demonstrate the expected SCK for the task during the third interview. He could not fully demonstrate the expected SCK as Karl. On the other hand, Kristin's belief about technology was instrumental during that time, and she demonstrated the expected SCK. Cameron's and Kristin's situation during the third interview led me to revisit the hypotheses I constructed at the beginning of the study: The level of technological expertise expected from a task might serve as another factor that affects the influence of belief about technology on SCK. The task during the first interview required more technological expertise and involvement, which necessitated substantive belief about technology for

SCK demonstration. However, technological expertise was not compulsory for the task during the third interview. PSTs only needed to run, view, stop and rerun the animation. Therefore, a PST who had an instrumental belief about technology could fully demonstrate the expected SCK during the third interview. To have a stronger claim about the link between teachers' beliefs about technology and their SCK development with technology, there is a need for a future research which focuses on the use of tasks requiring the same level of technological expertise from teachers.

Implications, Limitations and Future Research

One of the limitations of the study was the number of participants covered. I only interviewed six PSTs; experiences and beliefs of five PSTs were shared in this paper. The case study analysis allowed me to closely explore how beliefs of these teachers affected their specialized content knowledge. However, the same approach would not allow me to make any strong claims about pre-service education programs, but might provide direction for further analysis with quantitative studies. A quantitative study might focus on the analysis of SCK with a standardized test where the sample might be larger so that statistical inferences can be made. In addition, a valid and reliable survey to assess PSTs' beliefs about mathematics, teaching and technology might be used.

Another limitation of the study was its dependency on the tasks presented during interviews. These tasks were content dependent, where each PST's recalling abilities for this content might have affected their SCK they demonstrated. Moreover, the SCK they demonstrated only showed SCK for the specific content of the task which they dealt with. In a future qualitative study, I might focus on using tasks specialized for a unit in geometry rather than using tasks from different units. The complexity of these tasks should be similar for PSTs so that their difficulty would not be another factor mitigating their SCK.

In this study, I also looked at PSTs' beliefs isolated for mathematics, teaching and technology. However, the literature (Thompson, 1984) propounds that these beliefs are not isolated but connected where one type of belief under one domain (e.g. a type of belief about mathematics) might be linked with one type of belief under another belief domain (e.g. a type of belief about teaching). Future research might overcome this theoretical limitation by defining types of beliefs connected amongst different domains; and data would be analyzed according to these connected beliefs.

Overall, even though there were some limitations, the study showed that some type of beliefs about mathematics and technology might be influential for teachers to develop subject-matter knowledge specialized for the profession of teaching. Especially, Platonic beliefs about mathematics along with substantive beliefs about technology seemed to be guiding their geometry learning during their pre-service education. Regarding these findings from this study, pre-service education for middle school mathematics, and MAT programs should find ways to change these beliefs from instrumental to Platonic for mathematics, and from instrumental to substantive for technology. I think viewing mathematics as a bag of tools (i.e. the instrumental belief about mathematics) would be contradictory with the constructivist learning philosophies emphasized within education programs. In addition, learning mathematics conceptually might be difficult as long as prospective teachers view mathematics instrumentally.

Because of this, I could not say that the Platonic belief about mathematics is more correct than the instrumental belief about mathematics, but more necessary in order to achieve the aims of educational programs in terms of learning mathematics. Secondly, if we as teacher educators want mathematics teachers learn mathematics with technology the same as their students, then it might be quite difficult to reach this goal for those teachers who have an instrumental belief about technology. Therefore, I cannot claim that the substantive belief about technology is more correct than the instrumental belief about technology, but more necessary for mathematics teachers to be able to learn their content with technology. The graduate geometry course gave PSTs a chance to change these beliefs, but having a one semester experience to learn mathematics with a new instructional technology such as GSP might have been insufficient for them. Pre-service education both for undergraduate and graduate levels should give more chances of learning mathematics with technology so that their experiences would allow them to restructure their beliefs about mathematics and technology (Jonassen, 1995).

Appendix

Statements for Beliefs about Mathematics, Teaching and Technology

Statements for Beliefs about Mathematics

Which one(s) of the following would represent the nature of mathematics for you? You may choose more than one options.

Mathematics consists of certain definitions, procedures, methods that have to be acquired. Compilation of these tools forms the mathematics.	
Mathematics is about tricks and tactics to solve problems.	
Mathematics serves as a tool for other disciplines.	
Mathematics consists of concepts, procedures, definitions and the relationships among them.	
What makes mathematics special is its definite concepts and the connections among them. If you know the links among the concepts in mathematics, you could say that you also know mathematics.	
Mathematics is a discipline the same as physical sciences. It is a discovery more than a creation.	
People can rediscover mathematics like done by mathematicians the first time in the history.	
Mathematics is a human construction. It is not something to be received or transmitted.	
Mathematics is dynamic rather than static.	

Statements for Beliefs about Teaching

Which one(s) of the following would represent your perspective about teaching mathematics? You may choose more than one options.

Learners' background and interest are important to begin with in my teaching. The teacher should shape the instruction around learners' interest and knowledge, and guide them.	
Content is important in my teaching. Teacher should be in charge to help students understand concepts, their relationships and see the big picture.	
Content is important in my teaching. Teachers should be in charge to help students achieve in exams, tests, solve problems by themselves. Learning concepts in a short amount of time efficiently is important.	
Classroom management, assessment, plan and structure of the instruction and pedagogy are very important in my teaching. If the teacher knows how to manage the classroom, and is organized and meticulous in terms of the instruction and assessment, then teaching would go fluently.	

Statements for Beliefs about Technology

Which one(s) of the following statements do you agree with? You may be agree with more than one statement.

Technology should mainly be used to increase the efficiency of the instruction.	
Technology can sometimes inhibit students' understanding of math if not used appropriately.	
Technology inhibits students to learn basic mathematical skills.	
Technology has potential to enhance students' learning and understanding.	
Technology enables students' to be more creative, interpretive and analytical.	
Technology would change the math they are learning.	
Without technology, the math students are learning would be quite different.	

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CHAPTER SEVEN – CONCLUSION

In this chapter, I present conclusions for three research questions:

1) How does a Technology Integrated Geometry Course influence pre-service mathematics teachers' development of *SCK*?
2) How does a Technology Integrated Geometry Course influence pre-service mathematics teachers' development of *TCK*?
3) How do pre-service mathematics teachers' beliefs influence their SCK development?
    a. How are pre-service mathematics teachers' beliefs about *the discipline of mathematics* related to their SCK development?
    b. How are pre-service mathematics teachers' beliefs about *teaching mathematics* related to their SCK development?
    c. How are pre-service mathematics teachers' beliefs about *technology* related to their SCK development?

While I present findings for each of these research questions as three manuscripts in Chapter 4, 5, and 6, I follow a different structure within this chapter. Because Specialized Content Knowledge (SCK) (Ball, Thames, & Phelps, 2008) and Technological Content Knowledge (TCK) (Koehler & Mishra, 2005) became the two main constructs I investigated with this research, my conclusions pertaining to SCK are addressed in the first two subsections, and to TCK in the third subsection. My results demonstrate insight concerning the development of SCK in general as well as with the use of Geometer's Sketchpad (GSP). In the first subsection, I present my conclusions about the development of SCK independent of the use of GSP. Then, I underline important points about SCK development with GSP in the second subsection. In the final subsection, I discuss implications, limitations of my study and my future research routes regarding these limitations.

How to Develop Specialized Content Knowledge

Even though there have been several discussions by professionals and teacher educators on this topic (Leinhardt & Smith, 1985; Shulman, 1986; Fennema & Franke, 1992; CBMS, 2001; Ball, Lubienski & Mewborn, 2001), identification of subject-matter knowledge a mathematics teacher should have is still a challenge (Ferrini-Mundy & Findell, 2010). With new frameworks in mathematics education, mathematics teachers are expected to have a different type of mathematical knowledge than an engineer or a mathematician would have. SCK as a construct within Ball and her colleagues' MKT framework is one description of the mathematical knowledge a mathematics teacher should possess. In my study and with the development of the first and third manuscripts, I examined how teachers might develop this knowledge within a geometry content course and what factors were influential in the process of SCK development for pre-service teachers. The results of the study indicated that three factors seemed to be influential for SCK development: 1) Common Content Knowledge, 2) Supportive Contexts and Instructional Decisions, and 3) Teacher Beliefs about Mathematics and Teaching.

Common Content Knowledge

For this study, Common Content Knowledge (CCK) was defined as pre-service mathematics teachers' knowledge of mathematical facts, postulates, procedures and representations while solving a problem or dealing with a task. In a way, CCK was PSTs' knowledge base which allowed a PST to differentiate a correct answer from incorrect ones. Regarding its definition, SCK includes the knowledge needed beyond this differentiation, and includes mathematical knowledge regarding why an answer is correct or incorrect, what mathematical misconceptions may have influenced a student's incorrect answer, and how a representation could be mapped onto another representation.

Perhaps just by looking at this definition, CCK might be considered as a prerequisite for SCK development. If a mathematics teacher does not know mathematical facts and procedures while teaching a topic, it follows the teacher would struggle in answering why questions, questioning a student's unusual strategy, or examining a student's error. CCK would be prerequisite knowledge to develop SCK, especially for mathematical tasks requiring higher level thinking skills.

In Chapter 1, I discussed the necessity of developing content courses that would support the development of teachers' SCK, hypothesizing these courses could enable them to develop SCK before they began teaching. However, my results indicate these courses should also emphasize CCK development as much as SCK development. Without such consideration, these courses would only behave as methods courses given by mathematics departments which provide many opportunities in developing teaching strategies and methods to accelerate students' learning of mathematics, but fewer opportunities to strengthen their subject-matter knowledge. As much as SCK-related objectives, these content courses offered to prospective teachers should also address common content knowledge-related objectives that would be assessed at the end of the semester. I think such content courses with an emphasis on SCK development already exist, but have not been closely examined. With the recommendations in the *Mathematical Education for Teachers II* report (CBMS, 2012), mathematics teacher educators recognized that there was a need for mathematics courses that would bridge school mathematics with collegiate mathematics, and support teachers' SCK development in addition to CCK. After that, teacher educators developed such courses to meet this need. However, as this case study may indicate, these courses require an update of their objectives and priorities. I would re-examine these courses with an attention on CCK as much as SCK. In other words, if the priority would merely be on SCK with minimal attention to CCK included, these courses cannot achieve their objectives. If these courses are also to address the development of SCK however, additional attention also needs to be paid to the contexts and pedagogy of these courses.

Supportive Contexts and Instructional Decisions

This study showed utilizing PSTs' mathematical errors as opportunities to examine their ideas can enable them to develop SCK. SCK was partially defined as mathematical knowledge embedded within students' errors, unusual strategies and unexpected why questions. With respect to this definition, the instructor of the content course would be expected to identify moments where PSTs made mathematical errors or came up with different strategies as opportunities for the development of SCK. Those moments are not opportunities only for the PST who made the error, or used an unusual strategy, but also for other PSTs who can be involved with the discussion while making sense of their colleagues' mathematical reasoning. PSTs' own mathematical errors and their reasoning on their errors would enhance their mathematical knowledge about high school students' errors. Of course, in order for these opportunities to be leveraged, the instructors of these courses would also need to possess their own SCK. In this dissertation, especially in the first manuscript, the instructor utilized Kathleen's mathematical errors during class meetings as moments to unravel her thought process and capture what mathematics she used while ending up with such errors. During the second class meeting, where PSTs were sharing their ideas in constructing an angle bisector by using materials such as pins and strings, the instructor recognized that Kathleen's procedure was faulty, which led her to open a whole class discussion around her error. The discussion and the instructor's strategy to represent her procedure on GSP allowed other PSTs to notice that her procedure was not correct to construct an angle bisector, and later to comprehend why it was erroneous. The instructor made a similar instructional decision when a group of PSTs presented an unusual strategy to draw triangles having 2 square units on a geo-board. These examples from data indicate the importance of discussion around PSTs' alternative procedures and mathematical errors, and the instructor's questioning techniques so as to help other PSTs recognize mathematical reasoning while following an unusual procedure or having an error.

Bair and Rich (2011) include a similar component in their framework for SCK development: *Explaining Their Reasoning*. In order to develop SCK, teachers are expected to solve a problem first, explain their reasoning, and discuss possible students' errors. The ability to recognize possible students' errors and understand the mathematics embedded within these errors can be achieved by utilizing teachers' personal experiences in doing mathematics and making errors. Such opportunities provided by the course instructor can enable PSTs' SCK development. I also hypothesized and examined other teacher beliefs as individual factors affecting this phenomenon.

Teacher Beliefs about Mathematics and Teaching

In this study, there was not sufficient evidence to make an assumption for a potential connection between beliefs about mathematics and SCK development. All participants who demonstrated SCK through three interviews indicated the Platonic belief about mathematics, according to which, participants viewed mathematics as an exact and certain discipline that was embedded within the world.

I would theoretically assume that teachers should have both Platonic and problem solving beliefs about mathematics to develop their SCK. Teachers would focus on comprehending the concepts covered and grasping the relationships among if they recognize mathematics in such a structure. Secondly, it might be hard to find these relationships among concepts unless teachers view mathematics as a human construction. In this study, there were few instances where PSTs' views might have implicitly guided them for their SCK development. For example, Kristin was uncertain about the concept of height for an obtuse triangle. Her Platonist beliefs about mathematics might have helped her discover this concept while viewing the animation. However, these few instances were still not sufficient to make any claim on a relationship between this type of belief about mathematics and SCK development. There were other PSTs who held the same type of belief, but could not fully demonstrate SCK intended from the task.

Regarding Ernest's hierarchy of three beliefs about mathematics, I would have hoped PSTs would have held more of a problem-solving stance, but their prior experiences did not seem to support the development of this belief. In order for teachers to view mathematics as a human construct and a subject-matter dependent on language and communication, one would hypothesize that content courses need to be designed around mathematical activities which demonstrate mathematics as ideas open to interpretation and communication. The course utilized for this study was rich with whole class discussions and the instructor of the course presented mathematics as a subject that could be negotiated by participants. However, these experiences did not appear to be sufficient for PSTs to view mathematics as a human construct. The lack of significant change is predictable, as beliefs typically are developed and changed over a significant period of time (Anderson & Helms, 2001).

PSTs in this study also predominantly held a classroom-focused belief (Kuhs & Ball, 1986; cited in Thompson, 1992) toward teaching. The reason for the dominance of this belief can be attributed to the PSTs' lack of teaching experience. Their lack of experience in the classroom most likely caused their high level concerns to be focused on techniques of classroom management, assessment, and instructional preparation. Similar to their beliefs about mathematics, PSTs' beliefs about teaching did not change within the semester as well. van der Sandt (2007) related beliefs about mathematics to beliefs about teaching. For example, teachers having Platonic belief about mathematics (Ernest, 1989) would be expected to have a content-focused belief about teaching with an emphasis on conceptual understanding (Kuhs & Ball, 1986; cited in Thompson, 1992). However, this study did not support these results. One explanation to this situation could be the level of PSTs' honesty in sharing their beliefs. A participant might have considered that he should view mathematics in a certain way because it is a more *ideal* picture of mathematics, while displaying a different belief *in practice*. This dilemma in the identification of teachers' beliefs necessitates the usage of multiple scales and observational methods in order to differentiate ideal beliefs from real ones. In this study, I used participants' agreement with responses and statements as a way to determine their beliefs. However, these beliefs were not reflected on their actions during interviews. This discrepancy might violate the validity of findings about the relationship between beliefs and knowledge development. There is a need for a future research to overcome this limitation of this study and make valid claims for this relationship.

This study did not find enough evidence to conclude that teachers' beliefs about mathematics or teaching predicted their SCK development. One of the five PSTs who had a content-focused belief about teaching with an emphasis on conceptual understanding showed evidence of more SCK development compared to the other PSTs who held a classroom-focused belief about mathematics. Her beliefs might have allowed this PST to investigate concepts in mathematics and find the relationships among them more. However, this finding was not supported with other participants in the study. To examine this claim more in detail, there is a need for future research with more participants.

How to Develop Specialized Content Knowledge with Geometer's Sketchpad

In this study, technology was an important component of the course under investigation. One of the main hypotheses at the beginning of the study was that the technology, more specifically GSP, would enhance PSTs' SCK development. Findings from the first and third manuscript indicated the importance of 1) PSTs' openness to exploration; 2) their views about GSP; and 3) beliefs about technology as factors influencing SCK development.

Openness to Exploration and Viewing Geometer's Sketchpad as a Learner Partner

Another PST-related factor affecting SCK development was openness to exploration in mathematics. Findings presented in the first manuscript indicated that a teacher's openness to exploration with a given task determined the effectiveness of GSP on PSTs' SCK development.

GSP creates a digital environment where users can explore their conjectures and ideas. It can be approached as a new medium where geometry-based experiments can be conducted. If a teacher is not open to exploration, this new medium can change this habit through its dynamic feature and multiple representations. However, data also pointed out the necessity of teachers' preliminary views about GSP for the effectiveness on SCK development. If a PST views GSP as a tool for precise measurements and demonstration rather than a learning partner, then the role of GSP in SCK development might be limited. In other words, the effectiveness of GSP on SCK development is dependent on users' awareness about GSP's affordances and differences from other technologies. For some teachers, GSP can merely be viewed as a tool combining other technologies such as calculator, ruler and a drawing frame. Even though GSP can provide these affordances, the advantage of this technology emerges with the union of these affordances. Without viewing GSP as a lab unifying other technologies, GSP cannot be influential in SCK development. Regarding this, the instructor of a content course should also aim to help PSTs gain this view and work with GSP accordingly throughout the course. To do so, the instructor may have to raise disequilibrium within teachers' current beliefs by discussing and showing that different practices with GSP are favorable for their content development (Putnam & Borko, 1997). Experience with GSP as a geometry lab would also enable teachers to view it in this way (Habre & Grundmeier, 2007; Blanchard, Southerland, & Granger, 2009).

Teacher Beliefs about Technology

In the third manuscript, I examined PSTs' beliefs about technology (Chen, 2011) in order to see the link between teachers' beliefs and their SCK development. The same as the other belief types, PSTs did not change their beliefs about technology in one semester. I am aware that making any conclusion on beliefs and belief change is difficult because both beliefs and change are complex phenomena to examine. Because of this, if there was any belief change for these PSTs, the change might have been quite infinitesimal. One semester of experience with a new technology was not sufficient for them to change their beliefs (Pajares, 1992). The ages of the PSTs might have been another factor influencing their beliefs about technology; PSTs older than 35 tended to hold instrumental beliefs about technology (specifically GSP), while PSTs younger than 35 held substantive beliefs about technology. However, the strength of this claim needs to be tested with a larger sample size. Even though this difference existed among participants of the study throughout the semester, this difference was not reflected on their SCK development. I hypothesized at the beginning of the study that substantive beliefs about GSP would contribute to its effectiveness on SCK development. However, I had mixed findings with respect to this hypothesis. PSTs having substantive or instrumental beliefs about GSP demonstrated SCK during different tasks. In addition, having a substantive belief about technology did not always guarantee SCK development. The level of technological expertise expected from a task might have been another factor that affected the influence of belief about technology on SCK.

The following section focuses on my concluding remarks from the TCK component of the study by highlighting importance of results about how PSTs develop their TCK pertaining to the use of GSP within a geometry course.

How to Develop Technological Content Knowledge

TCK was defined in this study in two ways: technology knowledge about affordances of a specific instructional technology while dealing with a geometry task, and geometry knowledge to interpret a phenomenon demonstrated by an instructional technology. In the third manuscript, TCK was approached with respect to the first part of the definition.

Analytical Framework for TCK

The analysis of the data for this manuscript enabled me to construct an analytical framework consisting of four levels to assess teachers' TCK while using GSP. My main emphasis in the construction of the framework was to create a hypothetical learning trajectory for the ability to problem solve and model with GSP. The levels were also differentiated according to their accessibility to users within the software interface and to the unity of geometrical reasoning to make a better model and representation with GSP.

Analyses of PSTs' responses to the three tasks utilizing GSP indicated that the majority did not develop higher levels of TCK. These higher levels of TCK required the use of further affordances of GSP and the participating PSTs did not learn to use GSP to its full capacity. The major advantage of GSP was its affordance to represent geometrical phenomena dynamically. However, PSTs in this study utilized it as a measurement and drawing tool rather than as a lab to explore geometry dynamically. Olivera and Robutti (2007) discuss the difficulty of understanding geometrical concepts and constructing conjectures with dynamic geometry software if users do only use measurement and dragging affordances without having a clear picture about the phenomenon they observe. The limited duration of the study was compounded by the novelty of the technology, as this course was the first time they interacted with GSP. Considering the PSTs' development of TCK in this one semester study in one semester, I hypothesize the development of higher levels of TCK will require more sustained exposure to the instructional technology.

While improvement in technology knowledge is one necessity for TCK development, this study also underlines the importance of the quality of PSTs' geometry content knowledge. Sound understanding of geometric knowledge allowed some PSTs to utilize the dynamic aspect of GSP more effectively; and this in turn can result in development of TCK. Guerrero (2010) supported this finding and discussed that exploring geometry in depth with technology would present teachers with different mathematics content, which results in an expectation from teachers to be more confident in their content knowledge.

Implications and Future Research Direction

Specialized Content Knowledge

Overall, this study indicated important insights about the development of content courses designed to leverage technology such as GSP to facilitate PSTs' SCK development. Participants' experiences within the course and their interactions with tasks during three interviews suggested some preconditions need to be satisfied in order to support the effectiveness of GSP on SCK development: 1) CCK, 2) Supportive Contexts and Instructional Decisions (i.e. opportunities to examine other's errors, justifying ideas, the quality of tasks), 3) Teacher Beliefs (i.e. openness to exploration, viewing technology as a learner partner). Thus, while designing and delivering mathematics content courses offered for prospective teachers, it is important instructors: implement instructional strategies emphasizing PSTs' reasoning around their errors and unusual procedures, pose mathematical tasks requiring more exploration, and encourage PSTs to use technologies as learning partner rather than as new means for demonstration or measurement. Although I am aware that findings from this study cannot be representative for every teacher educator or course setting, this initial study certainly brought to light certain pedagogical choices shown to support the development of SCK.

One limitation of the study was the number of participants covered. The case study analysis allowed me to closely explore how beliefs of these teachers affected their SCK. However, the same approach would not allow me to make any strong claims about pre-service education programs, but might give some hints for further analysis with quantitative studies. A quantitative study might focus on the analysis of SCK with a standardized test where the sample might be larger so that statistical inferences can be made. In addition, a valid and reliable survey to assess PSTs' beliefs about mathematics, teaching and technology might be used.

Another limitation was the results' dependency on the tasks presented during interviews. These tasks were content dependent, where each PST's recalling abilities for this content might have affected his/her SCK they demonstrated. Moreover, the SCK they demonstrated only showed SCK for the specific content they dealt with the task. In a future qualitative study, I might focus on using tasks specialized for a unit in geometry rather than using tasks from different units. The complexity of these tasks should be similar for PSTs so that their difficulty would not be another factor moderating their SCK.

In this study, I also looked at PSTs' beliefs isolated for mathematics, teaching and technology. However, the literature (Thompson, 1984) propounds that these beliefs are not isolated but connected where one type of belief under one domain (e.g. a type of belief about mathematics) might be linked with one type of belief under another belief domain (e.g. a type of belief about teaching). The future research might overcome this theoretical limitation by defining types of beliefs connected amongst different domains; and data would be analyzed according to these connected beliefs.

Technological Content Knowledge

While Ball and her colleagues' Mathematical Knowledge for Teaching model (Ball, Thames, & Phelps, 2008) was used as the analytical framework for the first and third manuscript, I used the Technological Pedagogical Content Knowledge framework

(Koehler & Mishra, 2005) for the second manuscript to examine PSTs' TCK development within a geometry content course.

Regarding my findings from this study, in order to help PSTs to progress from superficial TCK toward expert TCK, teacher educators should give opportunities to work on "authentic curriculum problems for which technology-based solutions are collaboratively designed" (Voogt, Fisser, Roblin, Tondeur, & van Braak, 2013, p. 118). The authenticity of the problem should necessitate the use of technology so that PSTs can find a purpose in the use of technology while solving a problem. If the problem is not so complex that can be solved on a paper, then PSTs would never try new affordances of the technology, but follow the ones they have already know.

Koehler and Mishra's (2005) introduction of "learning technology by design" seems to be an effective way in order for teachers to develop their TCK. Learning technology by design can occur in two ways: 1) solving an authentic problem with technology by having a complex model constructed within a technology (in order to reach Preserved and Integration TCK), 2) preparing instructional products in order to meet instructional expectations and learning goals (in order to reach Expert TCK).

The course that I investigated allowed PSTs to work on authentic geometry problems, and the instructor asked them to use GSP to model these problems. In order to accelerate teachers' *learning technology by design*, PSTs might be introduced to the use of technologies as problem-solving tools as much as a new environment in which to represent a model. The teacher educator or the instructor of the course would be expected to be the expert in technology integration so that s/he can mentor PSTs in promoting how to learn mathematical content with technology differently. Secondly, these content courses offered to PSTs should also underline the identification of misconceptions, and seek technological representations in order to overcome these problems (Akkoç, 2011). Such an approach would both support teachers' content knowledge and TCK development at the same time.

In this study, I examined TCK development of PSTs who had changed their career from different fields into teaching. Because of that, my participants' background, previous experiences with mathematics, technology and pedagogy became one of the intervening factors for their development. These factors that I discussed for TCK development might not be applicable for PSTs who have had a stronger mathematics preparation. A future research study would investigate TCK development of PSTs who majored in mathematics and/or mathematics education.

Finally, I mainly focused on exploring rather than explaining PSTs' TCK development. This focus of my study allowed me to create an analytical framework out of my data analysis of PSTs' TCK development for appropriate tasks. The same framework might be beneficial to be used for future research in identifying PSTs' TCK and how its development was promoted or hindered in different course settings. As mentioned before, both the levels and the framework is not an end product, but a starting point in order to investigate TCK development. Regarding this, future research might also enable me to extend the framework and create further levels depending on the use of more complex affordances of a technology covered during a content course.

APPENDICES

Appendix A – Entrance Survey[4]

Thank you for taking time to complete this questionnaire. Please answer each question to the best of your knowledge. Your thoughtfulness and candid responses will be greatly appreciated. Your individual name or identification number will not at any time be associated with your responses. Your responses will be kept completely confidential and will not influence your course grade.

Demographic and Background Information

1. Your Name:
2. Age range
    a. 18-22
    b. 23-26
    c. 27-32
    d. 32+
3. What was your major prior to the MAT program?
4. Can you briefly tell us about your work experiences so far?
5. Did you take any other undergraduate and/or graduate geometry course before that one? Describe these courses. How did you perform in these courses? What did you find most difficult and/or easy?
6. Which instructional technologies do you know how to use? What technology have you used for mathematics teaching/learning (high school or college)? Have you had any experience with dynamic geometry software? If so, describe that experience.

[4] This survey was adapted from Schmidt, D. A., Baran, E., Thompson, A. D., Koehler, M. J., Mishra, P., & Shin, T. (2009). Survey of Preservice Teachers' Knowledge of Teaching and Technology. *Iowa State University, Iowa*.

Technology is a broad concept that can mean a lot of different things. For the purpose of this questionnaire, technology is referring to digital technology/technologies. That is, the digital tools we use such as computers, laptops, iPods, handhelds, interactive whiteboards, software programs, etc. Please answer all of the questions and if you are uncertain of or neutral about your response you may always select "Neither Agree or Disagree"

	Strongly Disagree	Disagree	Neither Agree or Disagree	Agree	Strongly Agree
TK (Technology Knowledge)					
1. I know how to solve my own technical problems.					
2. I can learn technology easily.					
3. I keep up with important new technologies.					
4. I frequently play around with technology.					
5. I know about a lot of different technologies.					
6. I have the technical skills I need to use technology.					
CK (Content Knowledge)					
Mathematics					
7. I have sufficient knowledge about mathematics.					
8. I can use a mathematical way of thinking.					
9. I have various ways and strategies of developing my understanding of mathematics.					
Social Studies					
10. I have sufficient knowledge about social studies.					
11. I can use a historical way of thinking.					
12. I have various ways and strategies of developing my understanding of social studies.					
Science					
13. I have sufficient knowledge about science.					
14. I can use a scientific way of thinking.					
15. I have various ways and strategies of developing my understanding of science.					

Literacy					
16. I have sufficient knowledge about literacy.					
17. I can use a literary way of thinking.					
18. I have various ways and strategies of developing my understanding of literacy.					
PK (Pedagogical Knowledge)					
19. I know how to assess student performance in a classroom.					
20. I can adapt my teaching based-upon what students currently understand or do not understand.					
21. I can adapt my teaching style to different learners.					
22. I can assess student learning in multiple ways.					
23. I can use a wide range of teaching approaches in a classroom setting.					
24. I am familiar with common student understandings and misconceptions.					
25. I know how to organize and maintain classroom management.					
PCK (Pedagogical Content Knowledge)					
26. I can select effective teaching approaches to guide student thinking and learning in mathematics.					
27. I can select effective teaching approaches to guide student thinking and learning in literacy.					
28. I can select effective teaching approaches to guide student thinking and learning in science.					
29. I can select effective teaching approaches to guide student thinking and learning in social studies.					

TCK (Technological Content Knowledge)

30. I know about technologies that I can use for understanding and doing mathematics.					
31. I know about technologies that I can use for understanding and doing literacy.					
32. I know about technologies that I can use for understanding and doing science.					
33. I know about technologies that I can use for understanding and doing social studies.					
TPK (Technological Pedagogical Knowledge)					
34. I can choose technologies that enhance the teaching approaches for a lesson.					
35. I can choose technologies that enhance students' learning for a lesson.					
36. My teacher education program has caused me to think more deeply about how technology could influence the teaching approaches I use in my classroom.					
37. I am thinking critically about how to use technology in my classroom.					
38. I can adapt the use of the technologies that I am learning about to different teaching activities.					
39. I can select technologies to use in my classroom that enhance what I teach, how I teach and what students learn.					
40. I can use strategies that combine content, technologies and teaching approaches that I learned about in my coursework in my classroom.					
41. I can provide leadership in helping others to coordinate the use of content, technologies and teaching approaches at my school and/or district.					
42. I can choose technologies that enhance the content for a lesson.					
TPACK (Technology Pedagogy and Content Knowledge)					

43. I can teach lessons that appropriately combine mathematics, technologies and teaching approaches.					
44. I can teach lessons that appropriately combine literacy, technologies and teaching approaches.					
45. I can teach lessons that appropriately combine science, technologies and teaching approaches.					
46. I can teach lessons that appropriately combine social studies, technologies and teaching approaches.					

Please complete this section by writing your responses in the boxes.

47. What do you see as the nature of mathematics? How do you think it is different from other subjects?

48. How should mathematics be taught? What supports do students need to learn mathematics?

49. What are your views on technology? What are the advantages or disadvantages of technology use in mathematics instruction?

50. Have you learned or taught a mathematical concept with dynamic geometry software (for example, Geometer's Sketchpad)? If so, please describe such experiences.

51. How might the use of Geometer's Sketchpad contribute to your understanding of geometry as a learner?

52. How might the use of Geometer's Sketchpad hinder your understanding of geometry as a learner?

Appendix B – Exit Survey[5]

Thank you for taking time to complete this questionnaire. Please answer each question to the best of your knowledge. Your thoughtfulness and candid responses will be greatly appreciated. Your individual name or identification number will not at any time be associated with your responses. Your responses will be kept completely confidential and will not influence your course grade.

Your Name:

[5] This survey was adapted from Schmidt, D. A., Baran, E., Thompson, A. D., Koehler, M. J., Mishra, P., & Shin, T. (2009). Survey of Preservice Teachers' Knowledge of Teaching and Technology. *Iowa State University, Iowa*.

Technology is a broad concept that can mean a lot of different things. For the purpose of this questionnaire, technology is referring to digital technology/technologies. That is, the digital tools we use such as computers, laptops, iPods, handhelds, interactive whiteboards, software programs, etc. Please answer all of the questions and if you are uncertain of or neutral about your response you may always select "Neither Agree or Disagree"

	Strongly Disagree	Disagree	Neither Agree or Disagree	Agree	Strongly Agree
TK (Technology Knowledge)					
1. I know how to solve my own technical problems.					
2. I can learn technology easily.					
3. I keep up with important new technologies.					
4. I frequently play around with technology.					
5. I know about a lot of different technologies.					
6. I have the technical skills I need to use technology.					
CK (Content Knowledge)					
Mathematics					
7. I have sufficient knowledge about mathematics.					
8. I can use a mathematical way of thinking.					
9. I have various ways and strategies of developing my understanding of mathematics.					
Social Studies					
10. I have sufficient knowledge about social studies.					
11. I can use a historical way of thinking.					
12. I have various ways and strategies of developing my understanding of social studies.					
Science					
13. I have sufficient knowledge about science.					
14. I can use a scientific way of thinking.					
15. I have various ways and strategies of developing my understanding of science.					

Literacy					
16. I have sufficient knowledge about literacy.					
17. I can use a literary way of thinking.					
18. I have various ways and strategies of developing my understanding of literacy.					
PK (Pedagogical Knowledge)					
19. I know how to assess student performance in a classroom.					
20. I can adapt my teaching based-upon what students currently understand or do not understand.					
21. I can adapt my teaching style to different learners.					
22. I can assess student learning in multiple ways.					
23. I can use a wide range of teaching approaches in a classroom setting.					
24. I am familiar with common student understandings and misconceptions.					
PCK (Pedagogical Content Knowledge)					
25. I can select effective teaching approaches to guide student thinking and learning in mathematics.					
26. I can select effective teaching approaches to guide student thinking and learning in literacy.					
27. I can select effective teaching approaches to guide student thinking and learning in science.					
28. I can select effective teaching approaches to guide student thinking and learning in social studies.					

	Strongly Disagree	Disagree	Neither Agree or Disagree	Agree	Strongly Agree
TCK (Technological Content Knowledge)					
29. I know about technologies that I can use for understanding and doing mathematics.					
30. I know about technologies that I can use for understanding and doing literacy.					
31. I know about technologies that I can use for understanding and doing science.					
32. I know about technologies that I can use for understanding and doing social studies.					
TPK (Technological Pedagogical Knowledge)					
33. I can choose technologies that enhance the teaching approaches for a lesson.					
34. I can choose technologies that enhance students' learning for a lesson.					
35. My teacher education program has caused me to think more deeply about how technology could influence the teaching approaches I use in my classroom.					
36. I am thinking critically about how to use technology in my classroom.					
37. I can adapt the use of the technologies that I am learning about to different teaching activities.					
38. I can select technologies to use in my classroom that enhance what I teach, how I teach and what students learn.					
39. I can use strategies that combine content, technologies and teaching approaches that I learned about in my coursework in my classroom.					
40. I can provide leadership in helping others to coordinate the use of content, technologies and teaching approaches at my school and/or district.					
41. I can choose technologies that enhance the content for a lesson.					

TPACK (Technology Pedagogy and Content Knowledge)					
42. I can teach lessons that appropriately combine mathematics, technologies and teaching approaches.					
43. I can teach lessons that appropriately combine literacy, technologies and teaching approaches.					
44. I can teach lessons that appropriately combine science, technologies and teaching approaches.					
45. I can teach lessons that appropriately combine social studies, technologies and teaching approaches.					

Please complete this section by writing your responses in the boxes.

46. What do you see as the nature of mathematics? How do you think it is different from other subjects?

47. How should mathematics be taught? What supports do students need to learn mathematics?

48. What are your views on technology? What are the advantages or disadvantages of technology use in mathematics instruction?

49. Have you learned or taught a mathematical concept with dynamic geometry software (for example, Geometer's Sketchpad)? If so, please describe such experiences.

50. How might the use of Geometer's Sketchpad contribute to your understanding of geometry as a learner?

51. How might the use of Geometer's Sketchpad hinder your understanding of geometry as a learner?

Appendix C – First Interview Protocol

PST Name:
Date:
Location:
Interviewer:
Start time/end time:

Task-Based Questions

Task A

1. Can you construct a square using only compass and straight edge?

2. *Reveal Handout 1*. Regarding the handout, can you complete the construction? *Probe*: What do you think the student would do next? Predict their method based on what is described.

What mathematics is behind student's strategy?

3. How would you construct a square using GSP? *Probe*: Is there any other way to perform the construction?

4. Can you construct a square using transformations? *Probe*: How would you construct a square using rotation? Do you see any connection between the representation given by the student and the one using rotation?

5. What are the advantages or disadvantages of using compass and straight edge compared to using GSP?

Experience-Based and Follow Up Questions

6. During the first class meeting, you bisected an angle by using pushpins and strings.

a. What was the method you followed to answer the question?

b. Did you see the flow within your procedure?

c. How did the demonstration with GSP by Nicole help your understanding?

7. During the first class meeting, you were supposed to construct geometrical principles such as bisecting an angle, or drawing a perpendicular line by using string and pushpins.

a. How did you proceed with the questions there? Did you recall your previous geometry knowledge from high school, or move with trial and error?

b. Were there any challenge for you to do these constructions with paper and pencil?

8. During the second class meeting, you viewed some postulates and made constructions out of them. One of the exercises allowed you to construct perpendicular bisector of a given line segment. Do you remember the method you followed for that? (*Show Artifact 2*)

a. Why do you think this method works to construct perpendicular bisector?

9. Did you ever use GSP for your ideas and conjectures about the perpendicular bisector construction? (*Show Artifact 2*)

a. If yes, please tell me more why you used GSP?

b. If not, why did you prefer to use paper/pencil, compass and straightedge?

10. Why do you think two intersecting circles let us to construct an equilateral triangle?

a. Your colleagues in that class answered the question as follows: “Two circles constructed the equilateral triangle because they are the same circles.” OR “Two circles constructed the equilateral triangle because they have the same radius.” Do you think two same circles always allow us to construct an equilateral triangle? Or do you think two circles having the same radius allow us to construct an equilateral triangle?

Belief Questions

11. Describe mathematics as a discipline in your own words? *Probe*: What do you see as the nature of mathematics? How do you think it is different from other subjects?

12. Which one(s) of the following would represent the nature of mathematics for you? You may choose more than one option.

Mathematics consists of certain definitions, procedures, methods that have to be acquired. Compilation of these tools forms the mathematics.	
Mathematics is about tricks and tactics to solve problems.	
Mathematics serves as a tool for other disciplines.	
Mathematics consists of concepts, procedures, definitions and the relationships among them.	
That makes mathematics special is its definite concepts and the connections among them. If you know the links among the concepts in mathematics, you could say that you also know mathematics.	
Mathematics is a discipline the same as physical sciences. It is a discovery more than a creation.	
People can rediscover mathematics like done by mathematicians the first time in the history.	
Mathematics is a human construction. It is not something to be received or transmitted.	
Mathematics is dynamic rather than static.	

Probe: Why did you choose these statements? Could you order the statements you chose with respect to their importance to you and your mathematics?

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13. How do you think mathematics should be taught? *Probe*: What supports do students need to learn mathematics?

14. Which one(s) of the following would represent your perspective about teaching mathematics? You may choose more than one option.

Learners' background and interest are important to begin with in my teaching. Teacher should shape the instruction around learners' interest and knowledge, and guide them.	
Content is important in my teaching. Teacher should be in charge to help students understand concepts, their relationships and see the big picture.	
Content is important in my teaching. Teachers should be in charge to help students achieve in exams, tests, solve problems by themselves. Learning concepts in a short amount of time efficiently is important.	
Classroom management, assessment, plan and structure of the instruction and pedagogy are very important in my teaching. If the teacher knows how to manage the classroom, and is organized and meticulous in terms of the instruction and assessment, then teaching would go fluently.	

Probe: Why did you choose these statements? Could you order the statements you chose with respect to their importance to you and your mathematics?

15. What are your views about technology? *Probe*: What are the advantages or disadvantages of technology use in mathematics instruction? What is the role of technology in teaching mathematics?

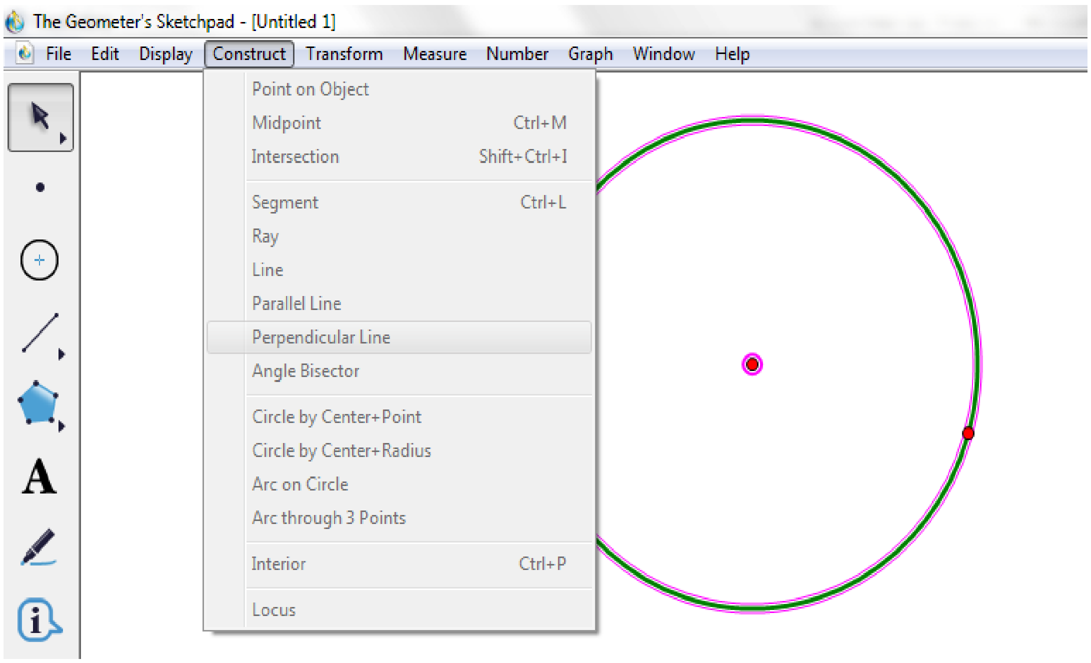
16. Which one(s) of the following statements do you agree with? You may agree with more than one statement.

Technology should mainly be used to increase the efficiency of the instruction.	
Technology can sometimes inhibit students' understanding of math if not used appropriately.	
Technology inhibits students to learn basic mathematical skills.	
Technology has potential to enhance students' learning and understanding.	
Technology enables students' to be more creative, interpretive and analytical.	
Technology would change the math they are learning.	
Without technology, the math students are learning would be quite different.	

Probe: Why did you choose these statements? Could you order the statements you chose with respect to their importance to you and your mathematics?

Handout 1

Susan claims she can construct a square by using the properties of a circle and lines having parallel or orthogonal properties. She began by constructing a circle by its center and around a point, and then constructing the radius.



Appendix D – Second Interview Protocol

PST Name:
Date:
Location:
Interviewer:
Start time/end time:

Task-Based Questions

Task B

1. *Reveal Handout 2. Open accompanying GSP document*. Regarding the story given in the handout, what should be the reason for the student's error? *Probe*: Do you think the error is math-related or technology-related? For each case, how would you guide the student to overcome the problem?

2. Would you say that you would not encounter such a student error if you don't use GSP for such teaching triangle inequality theory? *Probe*: Why or why not?

Experience-based and Follow Up Questions

What kind of relationship is there between internal and external angles of a triangle? Explore the applet! What do you see as the relationship?

3. What do you think did you learn from that course in terms of geometry so far? *Probe*: Any specific example from your experience within the course? Could you refer to any concept or topic in geometry?

4. Do you think GSP added to your knowledge or understanding of geometry?
Probe-a: If so, how? Any specific example from your experience within the course? Could you refer to any concept or topic in geometry? Why do you think GSP helped you understand?
Probe-b: If not, why do you think it did not add to your knowledge or understanding? Any specific example from your experience within the course?

5. What do you think did you learn from that course in terms of students' learning of geometry so far? *Probe*: Any specific example from your experience within the course? Can you give me an example of a student dealing with a task to show your point?
6. What do you think did you learn from that course in terms of how to teach geometry? *Probe*: Any specific example from your experience within the course? Can you give me an example of an instruction to show your point?

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7. In the shadow data gathering activity, several of you used GSP model that you constructed. Can you describe how the model was and how you used GSP during that time?

8. What kind of questions did you explore on the model, and how did you answer them?

9. On 24th of September, the class started with a discussion on a given homework about proportionality for triangles (*show the artifact*). For the exemplary problem, some of your colleagues thought that there are multiple answers. What do you think about that?

10. For the "Mirror Madness" activity, you created models on GSP. Can you describe it?

11. For "A Shadow of a Doubt" activity, the class was talking about three different ways to come up with the equation and proportionality. What was your method you mentioned? Why did you think that it was a SAS similarity?

12. In what ways do you think the use of GSP for mathematics instruction improved your ability to learn the geometry content so far? *Probe*: Can you give any specific example or topic from the course?

13. Would you say that technology changes the geometry content students learn?

Probe-a: If so, how? Can you think of an example of a geometry concept taught using GSP during one of the class meetings?

Probe-b: If not, why?

14. What can you do with GSP for a geometry class? *Probe*: Can you give me an example of an instruction to show your point from one of the class meetings? Could you achieve these without technology as well? How?

15. What are the limitations of GSP for a geometry class? *Probe*: For your teaching? How? Can you give me an example of an instruction to show your point?

Belief Questions

16. Could you describe mathematics as a discipline in your own words? *Probe*: What is the nature of mathematics according to you? How do you think it is different from other subjects?

17. Which one(s) of the following would represent the nature of mathematics for you? You may choose more than one options.

Mathematics consists of certain definitions, procedures, methods that have to be acquired. Compilation of these tools forms the mathematics.	
Mathematics is about tricks and tactics to solve problems.	
Mathematics serves as a tool for other disciplines.	
Mathematics consists of concepts, procedures, definitions and the relationships among them.	
That makes mathematics special is its definite concepts and the connections among them. If you know the links among the concepts in mathematics, you could say that you also know mathematics.	
Mathematics is a discipline the same as physical sciences. It is a discovery more than a creation.	
People can rediscover mathematics like done by mathematicians the first time in the history.	
Mathematics is a human construction. It is not something to be received or transmitted.	
Mathematics is dynamic rather than static.	

Probe: Why did you choose these statements? Could you order the statements you chose with respect to their importance to you and your mathematics?

18. How do you think mathematics should be taught? *Probe*: What supports students to learn mathematics? How would you describe teaching mathematics?

19. Which one(s) of the following would represent your perspective about teaching mathematics? You may choose more than one options.

Learners' background and interest are important to begin with in my teaching. Teacher should shape the instruction around learners' interest and knowledge, and guide them.	
Content is important in my teaching. Teacher should be in charge to help students understand concepts, their relationships and see the big picture.	
Content is important in my teaching. Teachers should be in charge to help students achieve in exams, tests, solve problems by themselves. Learning concepts in a short amount of time efficiently is important.	
Classroom management, assessment, plan and structure of the instruction and pedagogy are very important in my teaching. If the teacher knows how to manage the classroom, and is organized and meticulous in terms of the instruction and assessment, then teaching would go fluently.	

Probe: Why did you choose these statements? Could you order the statements you chose with respect to their importance to you and your mathematics?

---

20. What are your views about technology? *Probe*: What are the advantages or disadvantages of technology usage for mathematics instruction? What do you think the role of technology is in teaching mathematics?

21. Which one(s) of the following statements do you agree with? You may agree with more than one statement.

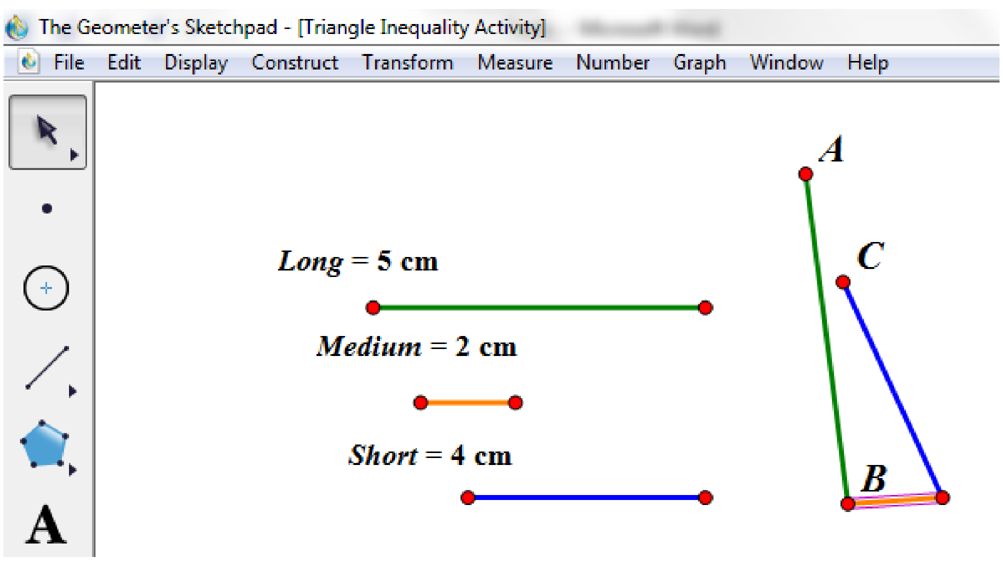
Technology should mainly be used to increase the efficiency of the instruction.	
Technology can sometimes inhibit students' understanding of math if not used appropriately.	
Technology inhibits students to learn basic mathematical skills.	
Technology has potential to enhance students' learning and understanding.	
Technology enables students' to be more creative, interpretive and analytical.	
Technology would change the math they are learning.	
Without technology, the math students are learning would be quite different.	

Probe: Why did you choose these statements? Could you order the statements you chose with respect to their importance to you and your mathematics?

Handout 2

You taught that, in any triangle, one side has to be smaller than or equal to the sum of two other sides, and larger than or equal to the absolute value of the difference between the other two sides ($|a-b| \leq c \leq (a+b)$).

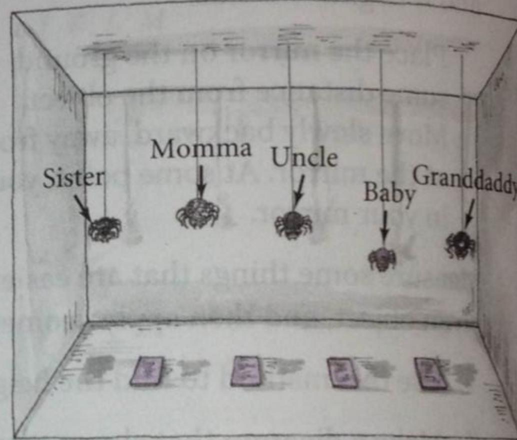
A student using GSP states that a triangle having sides measured 2, 4, 5 inches cannot be formed (*see the picture below*). However, regarding the triangle inequality, a triangle should be formed with these combinations. What should be the reason for the student's error?"



Mirror Madness

A family of spiders has found a bunch of mirrors on the ground. The spiders have been positioning themselves to see one another in the mirrors.

They are dangling in the order shown, although their distances from one another, their heights off the ground, and the positions of the mirrors are not necessarily drawn to scale.



Sister spider, who is 48 inches off the ground, can see Momma spider in a mirror that is on the ground between them. This mirror is 20 inches from the point directly below Sister spider and 30 inches from the point directly below Momma.

Momma spider can see Uncle spider in a mirror that is 10 inches from the point below Momma and 5 inches from the point below Uncle.

Uncle spider can see Baby spider in a mirror that is 8 inches from the point below Uncle and 6 inches from the point below Baby.

Finally, Baby spider can see Granddaddy spider in a mirror that is 12 inches from the point below Baby and 16 inches from the point below Granddaddy.

Your Task

Find the height of each spider. (You already know the height of Sister spider.) Show your equations and how you solved them.

Inventing Rules

Activity

In working with similar triangles, you often have to solve equations involving proportions.

Suppose one triangle has sides of lengths 6, 9, and 14. Suppose there is a similar triangle with shortest side of length 15. To find the longest side of the second triangle, represent it with x and find the value of x that satisfies this equation.

$$\frac{6}{15} = \frac{14}{x}$$

This is one of several possible equations for x.

Some equations of this kind are easier to solve than others. Sometimes the particular numbers involved suggest shortcuts that make them easy to solve.

In each equation, the letter x stands for an unknown number. Use any method you like to find what the number x stands for. Write down exactly how you do it.

Be sure to check your answers.

1. $\frac{x}{5} = 7$

2. $\frac{x}{6} = \frac{72}{24}$

3. $\frac{x}{8} = \frac{11}{4}$

4. $\frac{x}{7} = \frac{5}{3}$

5. $\frac{x+1}{3} = \frac{4}{6}$

6. $\frac{5}{13} = \frac{19}{x}$

7. $\frac{2}{x} = 6$

8. $\frac{9}{x} = \frac{x}{16}$

9. For each ratio in Question 4 and 8, draw a pair of similar triangles with side lengths that would create the ratios in the proportion.

10. Use any method you wish to solve for the unknown number in this proportion. Draw a pair of similar triangles that would reflect this proportion.

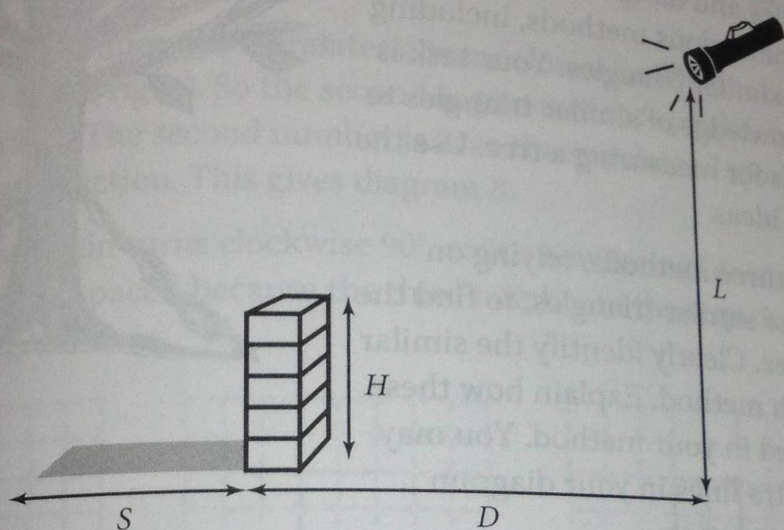
$$\frac{x+3}{7} = \frac{x}{3}$$

A Shadow of a Doubt

Activity

By now, you have probably developed a diagram similar to this one to represent the lamp shadow problem.

In terms of this diagram, the length of a shadow (S) depends on three things: the height of the light source (L), the height of the object casting the shadow (H), and the distance along the ground from that object to the light source (D).



1. What triangles do you see in the diagram? Which of them are similar? Why must these triangles be similar?
2. Use your knowledge of similar triangles to write an equation that expresses a relationship among the four variables.
3. Here is some information about a specific situation involving Shoshana, a lamppost, and her shadow. Shoshana is standing 9 feet from the lamppost. The light at the top of the lamppost is 12 feet high, and Shoshana is 5 feet tall. Use your equation from Question 2 to find the length of Shoshana's shadow.
4. Find the length of the shadow if $L = 11$, $H = 6$, and $D = 13$.
5. Write a description in words of how to find the length of a shadow when L, H, and D are given.

Appendix E – Third Interview Protocol

PST Name:
Date:
Location:
Interviewer:
Start time/end time:

Task-Based Questions

Task C

1. *Reveal Handout 3*. Are you aware of any relationships between the radius of a circle inscribed in a triangle, and the triangle's perimeter and area?

2. *Open the GSP document. Let the participant "play" with the document and the animation*. Regarding the animation within the GSP, what could be the relationship? *Probe*: how did you understand that? Could you explain it to me?

3. How else would you prove that 2S/P = r for that triangle? *Probe*: Can you prove this relationship another way?

4. Is there a difference between your proof and using the animation within GSP? *Probe*: How? Could you explain the difference?

Experience-Based and Follow Up Questions

5. How coordinates of a shape change when you reflect it across x-axis, y-axis, or for the graph of x=y axis? *Probe*: You can use anything if you want to explore and to be sure: paper and pencil or GSP.
   a. Is there anything new that you have learned about geometric transformations with GSP?
   b. Can you rotate a triangle around a point? Please demonstrate that on GSP?

6. On the activity that you used Geo-board, how did you start to solve the problem? The problem was asking to find any triangles having 2 units area and having a vertical line segment. *Probe*: Did you come up with any possibilities by using pegs and strings?

a. Cecilia and Shane demonstrated their methods on the board, which included several triangles having the same base and having different angles and direction. What do you think they were doing? What is the math behind her procedure/answer?

7. *Reveal the Puzzle Activity Sheet*. How do you think these puzzles prove the Pythagorean Theorem? How were you convinced about the theory with this puzzles? Can you explain it?

---

8. What do you think did you learn from that course in terms of geometry? *Probe*: Any specific example from your experience within the course? Could you refer to any concept or topic in geometry?

9. Do you think GSP added to your knowledge or understanding of geometry?

Probe-a: If so, how? Any specific example from your experience within the course? Could you refer to any concept or topic in geometry? Why do you think GSP helped you understand?
Probe-b: If not, why do you think it did not add to your knowledge or understanding? Any specific example from your experience within the course?

10. What do you think did you learn from that course in terms of students' learning of geometry? *Probe*: Any specific example from your experience within the course? Can you give me an example of a student dealing with a task to show your point?

11. What do you think did you learn from that course in terms of how to teach geometry? *Probe*: Any specific example from your experience within the course? Can you give me an example of an instruction to show your point?

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12. In what ways do you think the use of GSP for mathematics instruction improved your ability to learn the geometry content? *Probe*: Can you give any specific example or topic from the course?

13. Would you say that technology changes the geometry content students learn?

Probe-a: If so, how? Can you think of an example of a geometry concept taught using GSP during one of the class meetings?

Probe-b: If not, why?

14. What can you do with GSP for a geometry class? Probe: Can you give me an example of an instruction to show your point from one of the class meetings? Could you achieve these without technology as well? How?

15. What are the limitations of GSP for a geometry class? *Probe*: For your teaching? How? Can you give me an example of an instruction to show your point?

Belief Questions

16. Could you describe mathematics as a discipline in your own words? *Probe*: What is the nature of mathematics according to you? How do you think it is different from other subjects?

17. Which one(s) of the following would represent the nature of mathematics for you? You may choose more than one options.

Mathematics consists of certain definitions, procedures, methods that have to be acquired. Compilation of these tools forms the mathematics.	
Mathematics is about tricks and tactics to solve problems.	
Mathematics serves as a tool for other disciplines.	
Mathematics consists of concepts, procedures, definitions and the relationships among them.	
That makes mathematics special is its definite concepts and the connections among them. If you know the links among the concepts in mathematics, you could say that you also know mathematics.	
Mathematics is a discipline the same as physical sciences. It is a discovery more than a creation.	
People can rediscover mathematics like done by mathematicians the first time in the history.	
Mathematics is a human construction. It is not something to be received or transmitted.	
Mathematics is dynamic rather than static.	

Probe: Why did you choose these statements? Could you order the statements you chose with respect to their importance to you and your mathematics?

18. How do you think mathematics should be taught? *Probe*: What supports students to learn mathematics? How would you describe teaching mathematics?

19. Which one(s) of the following would represent your perspective about teaching mathematics? You may choose more than one options.

Learners' background and interest are important to begin with in my teaching. Teacher should shape the instruction around learners' interest and knowledge, and guide them.	
Content is important in my teaching. Teacher should be in charge to help students understand concepts, their relationships and see the big picture.	
Content is important in my teaching. Teachers should be in charge to help students achieve in exams, tests, solve problems by themselves. Learning concepts in a short amount of time efficiently is important.	
Classroom management, assessment, plan and structure of the instruction and pedagogy are very important in my teaching. If the teacher knows how to manage the classroom, and is organized and meticulous in terms of the instruction and assessment, then teaching would go fluently.	

Probe: Why did you choose these statements? Could you order the statements you chose with respect to their importance to you and your mathematics?

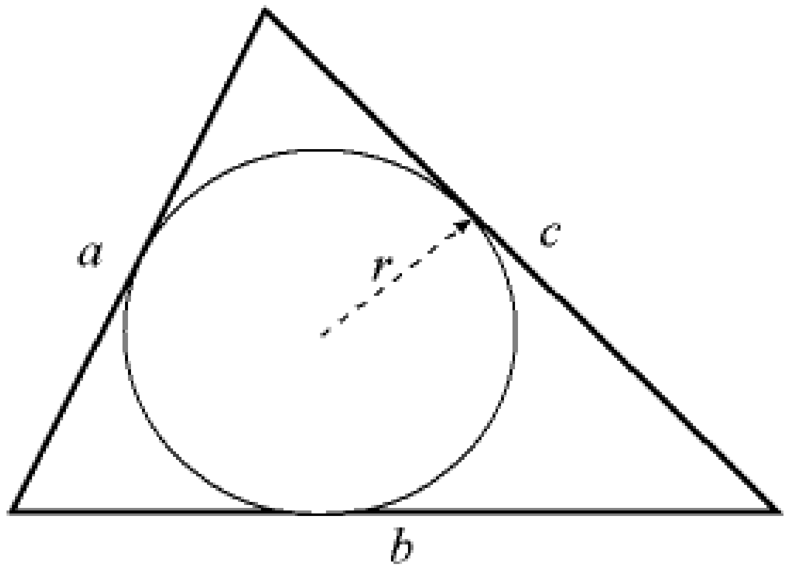
20. What are your views about technology? *Probe*: What are the advantages or disadvantages of technology usage for mathematics instruction? What do you think the role of technology is in teaching mathematics?

21. Which one(s) of the following statements do you agree with? You may be agree with more than one statement.

Technology should mainly be used to increase the efficiency of the instruction.	
Technology can sometimes inhibit students' understanding of math if not used appropriately.	
Technology inhibits students to learn basic mathematical skills.	
Technology has potential to enhance students' learning and understanding.	
Technology enables students' to be more creative, interpretive and analytical.	
Technology would change the math they are learning.	
Without technology, the math students are learning would be quite different.	

Probe: Why did you choose these statements? Could you order the statements you chose with respect to their importance to you and your mathematics?

Handout 3



Appendix F – Observation Protocol

Date:
Start time/End time:
Observer:
Instructor Name:
Number of Students:
Number of Female/Male:
Lesson Title:

Use the table in the following page according to the options below

Who majorly use the technology:	a) Not Evident b) Instructor c) Students d) Both
Which way the technology is used:	a) Not Evident / To Present Info b) To Demonstrate Student Task c) For Visualization a Concept d) For Grading, Attendance, Material Management
Instructional method:	a) Teacher Leading b) Students Leading c) Other
Interaction:	a) Whole class b) Small group c) Individual

Duration	Instruction Method	Interaction	Technology Used	Who Use Technology	Method for Technology Usage	Description/Comment
0-15 Minutes						
15-30 Minutes						
30-45 Minutes						
45-60 Minutes						
60-75 Minutes						
75-90 Minutes						

90-105 Minutes						
105-120 Minutes						
120-135 Minutes						
135-150 Minutes						
150-165 Minutes						

1. Describe if teachers posed any *why questions* about a mathematical procedure, concept or representation. Please narrate such instance here as the description of any interaction among teachers or between teachers and the instructor:

Describe what *the role of technology* within this instance of while investigating the why questions.

2. Describe if teachers were presented with *different representations* for any concept and whether and how they linked the representations. Please narrate such instance here as the description of any interaction among teachers or between teachers and the instructor:

Describe *the role of technology* within this instance of investigating multiple representations.

3. Describe if any teacher encountered with a *mathematical error* and whether and how other teachers and the instructor tried to understand the mathematics behind the error. Please narrate such instance here as the description of any interaction among teachers or between teachers and the instructor:

Describe what *the role of technology* within this instance of while investigating the mathematical error.

4. Describe if any teacher posed any unusual *mathematical procedure* for a given concept or problem and whether and how other teachers and the instructor tried to understand the mathematics behind proposed procedure. Please narrate such instance here as the description of any interaction among teachers or between teachers and the instructor:

Describe what *the role of technology* within this instance of while investigating the unusual procedure.

5. Describe if teachers made any prediction in terms of *possible students' understanding or confusion* while dealing with the concept covered. Please narrate such instance here as the description of any interaction among teachers or between teachers and the instructor:

Describe what *the role of technology* within this instance of while investigating the unusual procedure.

6. Describe if teachers made any discussion in terms of its links to *possible future instructional usage,* or teaching. Please narrate such instance or classroom scenario here as the description of any interaction among teachers or between teachers and the instructor:

Describe what *the role of technology* within this instance of while investigating the unusual procedure.

7. What is the specific benefit of *technology to support content*? What are the students and teacher doing that they could not have done without it? Please narrate such instance here as the description of any interaction among teachers or between teachers and the instructor:

8. What is the specific benefit of *technology to support pedagogy*? What are the students and teacher doing that they could not have done without it? Please narrate such instance here as the description of any interaction among teachers or between teachers and the instructor:

9. Describe if any teacher shared his/her opinion about the nature of mathematics/geometry.

10. Describe if any teacher shared his/her opinion about teaching mathematics/geometry.

11. Describe if any teacher shared his/her opinion about technology or teaching with technology.

Appendix G – The Syllabus of the Course Observed

Clemson University at the University Center Greenville

MTHS 7090-400: Geometry for the Middle Grades – Fall 2013

Tuesdays 5:00-7:45pm – MAT Suite Room 714

Professor: Dr. Nicole Bannister **Email:** nbannis@clemson.edu (***preferred***) **Twitter**: @CUMATMathDrB

Office Phone: (864) 250 - 6709 **Office Location:** UCG – Clemson MAT Suite D6 – Office D

UCG Office Hours: Mondays 3-5pm; Tuesdays 7:45-8:45pm; other times by appointment

Main Campus Office Hours (Martin Hall Basement Office O-1): Tuesdays, Thursdays 9:45-10:45am; Thursdays 12:45-1:45pm

This syllabus is subject to change at the discretion of the instructor.

Course Overview

This course centers on making sense of geometry topics fundamental to the middle grades curriculum. We will focus on:

> Hands-on approach to constructions with straight-edge and compass; polygons including tessellations and polyhedra; symmetry and transformational geometry; coordinate geometry measurement with dimensional analysis; perspective drawing and related topics; history of geometry; reasoning and informal proof with congruence; and computer software, calculator use, and Internet.

We will learn mathematics by doing mathematics together, and then connecting this mathematics to the Common Core State Standards (CCSS) and the courses that you teach. A variety of teaching methods, "high-leverage" practices, and assessment strategies will be modeled and used in the course.

Learning Outcomes:

1. The student will be able to describe the geometry content of typical middle grades mathematics courses and identify the core underlying mathematical ideas of these courses.
2. The student will be able to describe the similarities and differences in course standards for middle grades geometry.
3. The student will be able to use dynamic geometric software flexibly and fluidly during problem solving tasks.
4. The student will be able to use manipulatives during problem solving tasks and identify their appropriate use in secondary geometry classrooms.
5. The student will be able to explain, justify, and write proofs related to course content.

Learner Expectations:

- **Citizenship:** Learning is a social process. Participating actively and fully in classroom tasks is vital to this class. Your participation in our class activities is important not only for your own learning but also the learning of others. Sharing ideas and questions with the group, as well as responding to those of your classmates, are critical to our work together. As a teacher, you need to do more than understand your own thinking – you have to listen to others' thinking, figure out what others are saying, and determine whether and how they make sense. In our class, the "others" will be your colleagues. So listening to and interacting with them is explicitly to help you develop dispositions and skills that matter for teaching. We understand that some people are more comfortable than others with verbal participation, while others will be challenged to listen. This is a chance for us to hold each other accountable for developing the kind of learning community we hope to foster for our students, one that is safe, equitable, and in which everyone learns through various forms of participation. Please note that you are expected to be prepared for all class meetings with all materials and assignments completed. Moreoever, this class was carefully designed and is being implemented purposefully. As such, I expect you to talk to me about our discussions, assignments, readings, course design, teaching strategies, and other areas of interest sparked by our work together.
- **Attendance Policy**: Due to the participatory nature of this course, students are exepcted to attend every class and be an active participant in the classroom practices. Students are allowed one absence without penalty to the course grade, and a make-up essay will be given in place of the missed class. A second (unexcused) absence will result in a 10% reduction of the final grade. A third absence will result in an additional 10% reduction in the final grade. Any student missing more than three classes will be asked to take this class during a semester more conductive to their active involvement. For each absence, excused or unexcused, the student will complete a make-up assignment consisting of constructed-responses problem-solving opportunities. Consult Dr. Bannister for further details. In the event of an absence or any nature, students are still responsible for handing in work on the assigned due date unless cleared by instructor AHEAD of time. Late work loses 5 points a day. Students must provide the instructor with proper documentation for university-sanctioned absences. Three instances of tardiness of 15 minutes or more or three instances of leaving 15 minutes or more before the end of class or a combination of the two will count as one absence.
- **Instructor Lateness**: Students at Clemson are expected to wait 15 minutes in the event that the instructor is late. If, after 15 minutes, the instructor *or appropriate instructions have not arrived*, students may leave without incurring a class absence. Have someone in the class call my mobile phone (425-442-8549) to determine the problem. Also check your email for possible updates or instructions.
- **Technology and Equipment Requirements**: You are expected to check your Clemson University email on a daily basis. You are expected to use proper email etiquette when sending messages to the instructor and your peers. You are expected to have regular Internet access and use our classroom Blackboard space. You are expected to use a computer for word processing and bring a flashdrive with you to class. You must use Geometer's Sketchpad software for assignments that require dynamic geometry software. It will be helpful to bring your laptop with you to our class meetings.

Required Textbooks & Materials

Required Materials:

- Compass
- Ruler and/or Straightedge
- Graph paper
- Colored pencils or pens
- Geometer's Sketchpad (Version 5) on the laptop you bring with you to each class.
    - Downloadable from: https://www.mheonline.com/program/view/2/16/2647/00000SPAD#program

- Choose one: A 1-year student license for about \$10 and a non-expiring license for about \$70.
- There is also a related GSP iPad app, though it will only let you work with sketches made on your regular computer.

Required Textbooks:

- Interactive Mathematics Program (IMP) Unit Books (2nd edition):
  - Shadows (Year 1): ISBN: 978-1-55953-999-9
  - Do Bees Build it Best (Year 2): ISBN: 978-1-60440-031-1
  - Download copies of the teacher's editions here: http://impmoodle.its-about-time.com/
- Looking for Pythagoras – Connected Math Project (CMP2) – Student Unit Book – ISBN 0133661504

Blackboard Website

http://bb.clemson.edu – Follow links to our section of MTHS 7090 in Blackboard. You are responsible for checking this website and your university email account (userid@clemson.edu) on a regular basis for announcements and class materials. This course website houses all course materials: the course syllabus, course schedule, announcements, links to group activities and keys, and student grades. *Additional textbook readings and assignments will be distributed in class and/or on Blackboard.*

General Information

Attendance: I strongly encourage you to attend class regularly. However, if you must miss class YOU are responsible for the notes and assignments you missed. I will be taking attendance every day for my records. Students with more than 3 absences are subject to being dropped from the course. You must provide me with proper documentation for university-sanctioned absences. You may use the electronic notice system to inform me of non-university sanctioned absences. If you do have an excused absence the student should work with me (the instructor) to schedule making up group activities. If I do not arrive in the classroom within 15 minutes after the scheduled start time, class is dismissed for the day.

Email: You are free to email me anytime. Emails will generally be answered within 24 hours. Know that any email sent to me after 5pm is not guaranteed to be answered before the next business day. Please indicate your name and which course (Mthsc 408) you are taking in the email or the subject line.

Evaluations: At the end of the semester you are encouraged to fill out the instructor/course evaluations. Often there are some additional points added OR assignments dropped if a percentage of the class fills out the evaluations. More detail on this toward the end of the semester.

Special Accommodations: If you have a letter stating specific testing accommodations to which you are entitled, please email a copy to me as soon as possible so that we can make arrangements. See Student Disability Services – Student

Guide: http://www.clemson.edu/sds/student_guide.html

Academic Integrity: Students are expected to adhere to the following official Clemson academic integrity statement. As members of the Clemson University community, we have inherited Thomas Green Clemson's vision of this institution as a "high seminary of learning." Fundamental to this vision is a mutual commitment to truthfulness, honor, and responsibility, without which we cannot earn the trust and respect of others. Furthermore, we recognize that academic dishonesty detracts from the value of a Clemson degree. Therefore, we shall not tolerate lying, cheating, or stealing in any form.

Course Framework

These framing ideas originate from well-documented issues in math education. I thank many colleagues for influencing my thinking and for sharing their course ideas with us.

- **Think Deeply of Simple Things**

What mathematics should students learn in middle school? How should they learn it? How we answer these simple-sounding questions has significant consequences for your future students and the mathematics they have the opportunity to learn. This important intellectual work is inherently messy, and requires that we understand essential content, solve authentic problems, grapple with important dilemmas, use the language of mathematics, and think deeply about how to engage our students in this work.

- **Make Sense of a Complex Problem: *Who* Gets to Learn *What* Math?**

As it turns out, who you are strongly predicts what you get to learn in math class. A large body of literature documents the fact that mathematics consistently plays a gatekeeper role for many students (Moses, 2001; NRC, 1989; Schoenfeld, 1988, 2002). As Schoenfeld (2002) explains, "course work in mathematics has traditionally been a gateway to technological literacy and to higher education" (p. 13). However, for the large numbers of students who fail or leave mathematics course work, mathematics has instead served as a gatekeeper from higher education and economic access (Moses, 2001; Schoenfeld, 2002). Civil rights leader Bob Moses (2001) argues,

> Today…the most urgent social issue affecting poor people and people of color is economic access. In today's world, economic access and full citizenship depend crucially on math and science literacy. I believe that the absence of math literacy in urban and rural communities throughout this country is an issue as urgent as the lack of Black voters in Mississippi was in 1961. (p. 5)

As an NRC report put it bluntly, "More than any other subject, mathematics filters students out of programs leading to scientific and professional careers […] Mathematics is the worst curricular villain in driving students to failure in school" (1989, p. 7). Making matters worse, disproportionate numbers of historically marginalized students compose this group, meaning that working-class students, students of color, students who have beliefs that counter mainstream ideas, students who are not proficient in English, or students who do not meet the dominant cultural definitions of "normal" are marginalized in their mathematics classes more than their peers (Gutierrez, 2002; 2012). These harsh realities have renewed interest and urgency in understanding and creating *equitable mathematics classrooms*, which I follow Gutierrez (2002; 2008; 2012) and characterize as spaces where we cannot "predict mathematics achievement and participation based solely on student characteristics such as race, class, ethnicity, sex, beliefs, and proficiency in the dominant language" (Gutierrez, 2002, p. 153).

- **Learn What *WE* Can Do**

Words like *equity* and *democracy* evoke our most fervent hopes for education, prompting us to imagine how schools, just possibly, might be in the best of worlds. Before we follow through on our *equity* and *democracy* impulses, we get to tune our ideas about practices and responsibilities to the real-life complexities of teaching and learning. With *equity* and *democracy* in mind, a goal of this course is to encourage a growth in our understanding and appreciation of this complexity – and not simply the complexity of the classroom, but the terrifically complex relationship between classroom life and the rest of the world.

We define heterogeneous classrooms as settings in which students have a wide range of previous academic achievement and varying levels of oral and written proficiency in the language of instruction. Ensuring that all students in heterogeneous classrooms have access to academically challenging curricula and to equal-status participation, and can successfully demonstrate their understandings and skills is a fundamental pedagogical objective. We will learn how to build equitable mathematics classrooms where students engage in intellectually rigorous and linguistically rich learning tasks. For such classrooms, groupwork and cooperative or collaborative learning are highly recommended and well-documented instructional strategies.

- **Revise How We Think And Talk About Kids**

We strive for responsible descriptions of what is really going on in *real schools* with *real people* in them. This means that we have to interrogate our conventional understandings, including the conventional concepts we use to think and talk about school. We proceed as if we should revise our personal dictionary of terms for kids, learning, community, intelligence, and so on, as part of our work for equity and democracy. Perhaps we need to figure out an altogether new vocabulary if we are to get our way.

Of course, we are neither the first nor the only people to take on such a project. In this course, we use the work that others have already done to help us think, talk, and act collectively in more responsible ways. Rather than direct our readings and discussion exclusively toward the search for immediate "best practices," – always as unsatisfying as they are simplistic — we will work with each other to identify ideas that are useful and/or problematic in some long run: for the next year, yes, but for ten years from now as well. Overall, the course ought to be good, hard fun.

- **Connect This Work To Your Future Classroom Practices: Learn To Think and Act Like A Math Teacher**

Put out a nice meal, and people will know what to do with it. Different people in different ways for different versions of a meal, of course, but the regularities are visible. It is always possible, with careful attention, to pick up and carry out how the members of some group expect people to proceed. It is a matter of manners. The same for educational problems. Throw a topic on the table – say, tracking, level playing fields, bell curves, abilities and disabilities, race and social class, caring and fairness (these two pitted against reality) – and people will go at them with great regularity. Each problem will be taken seriously, opposing sides will get defined, policies and reforms urged, and moral fibers questioned and asserted. At the end of the day, the year, and even the generation, if all goes well, the changes will have been subtle, and the overall production and distribution of cultural and economic resources (whether for the dinner table or for schools) may look terribly like they always had. As teachers, our victories will be small, local, and, to that extent, heroically important.

Our course is designed to contribute to the subtle changes by interfering with our – yes, our – knee-jerk responses to the ways educational problems are usually defined. Our first goal is to transform the discussion of race, class, gender, sexual orientation, and how this relates to students' learning of mathematics. A second goal is directed to how you think about the problems that develop right in front of your face in your own classrooms filled with children. What are you going to do with the first child you don't like, or want to give up on, or find one hundred reasons to forget? You will not be alone in these problems. They happen everywhere called educative in American society. There is a way they are the problems of the children you are asked to save and nurture, a way they are your problem, and a way they belong to everyone. How are you going to worry about them? If you leave this course with a different way – any second way, please – of thinking and talking about and responding to our own engagement and investment in the production of the very troubles we are trying to solve, we will have made a contribution to subtle change. The third effort for change, a third goal, is to worry about these problems at the level of classroom practice.

Grading and Assignments

Point Distribution for Assignments:		**Grade Calculation:**	
Group Investigations:	25%	A:	90-100%
Major Individual Assignments:	25%	B:	80-89%
Minor Individual Assignments:	20%	C:	70-79%
Midterm:	15%	D:	60-69%
Final Exam:	15%	F:	< 60%

Group Investigations: You will complete instructional activities in small groups during most classes alongside a group "product" that is turned in on behalf of the group. Examples include group problem solving tasks and CCSS middle grades geometry sense making tasks. These assignments will be graded on clarity, correctness, and completeness. You are expected to participate actively and full in classroom tasks. Participation is valuable to your learning as well as the learning of others. Failure to actively participate in groupwork and/or class discussion will be reflected in your grade.

Major Individual Assignments: You will complete individual assignments that provide evidence of your learning of core course content. Examples may include unit portfolios, writing 2-3 inquiry-based geometry tasks, and larger problem solving tasks.

Minor Individual Assignments: You will complete individual assignments that provide evidence of your learning of core course content. Examples may include homework, GSP labs, geometry pre/post-surveys, reflective writing assignments, and smaller problem solving tasks.

Midterm: In the middle of the course you will be asked to complete a take-home midterm exam. Details will be presented at a later date. The midterm will be out of 100 points. (There will be a 15% penalty for every day the midterm is late – this includes weekends)

Final Exam/Project: At the end of the course you will be asked to complete a final exam/project. Details will be presented at a later date. The final will be out of 100 points. The Final Exam is comprehensive.

Tentative Coursework Schedule

Prospective Units:

1. **Constructions:** History of Early Geometry; Ruler & Straightedge; Origami; Dynamic Geometry Software
2. **Shadows:** Similarity and Congruence**;** Proportional Reasoning and the Algebra of Proportions**;** Polygons and Angles**;** Logic and Proof**;** Right Triangles and Trigonometry**;** Experiments and Data Analysis**;** Mathematical Modeling
3. **Do Bees Build It Best?** Area**;** The Pythagorean Theorem (reference *Looking for Pythagoras*)**;** Surface Area and Volume
4. **Circle Geometry:** Coordinate Geometry**;** Circles**;** Synthetic Geometry**;** Algebra**;** Logic & Proof
5. **Transformations:** translations, rotations, and reflections; connections to matrices

Meeting Schedule & Due Dates	Notes & Assignments
Aug 27	**Geometry Pre-Survey**
Sept 3	
Sept 10	
Sept 17	
Sept 24	**Take Home Midterm Assigned**
Oct 1	**Take Home Midterm Due**
Oct 8	
Oct 15	**No Class Meeting: Clemson University Fall Break**
Oct 22	***Note: SCCTM Fall Conference: October 24-25, Greenville, SC – http://scctm.org/***
Oct 29	
Nov 5	
Nov 12	
Nov 19	**Geometry Post-Survey**
Nov 26	**No Class Meeting: Your choice: Payback Day for SCCTM or Online Assignment**
Dec 3	**Final Exam**

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